Disagreement about Inflation and the Yield Curve
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Outline

- Why care about the term structure (real and nominal)
- Bond yields and the SDF
- Real yields and inflation
- Main theoretical result
- Comments
Why care about the term structure of interest rates?

Term structure tells us

- nominal rates at which the government can borrow and the rate at which investors are willing lend to the government
- crucial for investment decisions of firms, savings decisions of consumers, and policy decisions.

Can use yield spreads for forecasting

- future short yields (Campbell and Shiller, 1991; Cochrane and Piazzesi, 2005; Fama and Bliss, 1987)
- real activity (Ang et al., 2006; Estrella and Hardouvelis, 1991; Hamilton and Kim, 2002; Harvey, 1988) and inflation (Fama, 1990; Mishkin, 1990)

Monetary policy:

- central bank can move the short end of the yield curve, but long term yields important for aggregate demand, e.g. long term mortgage rates
- term structure model: short term yields $\rightarrow$ long term yields

Debt policy:

- how much to issue and when.
- how does this impact term structure?
This paper

- Inflation impacts real yields
- Why?
  - Differences in beliefs about inflation lead to consumption risk sharing across agents.
  - Consumption allocations impacted by differences in beliefs about inflation.
  - Real SDF impacted by differences in beliefs about inflation.
Bond yields and the SDF

- real and nominal SDF

\[
\overset{\text{nominal SDF}}{\xi^*(t)} = \frac{\overset{\text{real SDF}}{\xi(t)}}{\overset{\text{price level}}{\pi(t)}}
\]

- nominal zero cpn bond

\[
P(t, T) = E_t \left[ \frac{\xi(T)}{\xi(t)} \frac{\pi(t)}{\pi(T)} \right] = E_t \left[ \frac{\xi(T)}{\xi(t)} \right] E_t \left[ \frac{\pi(t)}{\pi(T)} \right] + Cov_t \left[ \frac{\xi(T)}{\xi(t)}, \frac{\pi(t)}{\pi(T)} \right]
\]

\[
= B(t, T) E_t \left[ e^{-I(t, T)} \right] + Cov_t \left[ \frac{\xi(T)}{\xi(t)}, e^{-I(t, T)} \right]
\]

real zero cpn bond  

inflation risk premium
Inflation and bond prices

\[ P(t, T) = B(t, T) \underbrace{E_t \left[ e^{-I(t, T)} \right]}_{\text{real zero cpn bond}} + Cov_t \left[ \frac{\xi(T)}{\xi(t)}, e^{-I(t,T)} \right] \]  

(1)

- **Implications**
  - High expected inflation → lower nominal bond price
  - future inflation high in bad states → lower nominal bond price
Empirical results in this paper: differences in beliefs about inflation
- Greater differences in beliefs: higher nominal yield, higher vol
- Greater differences in beliefs: higher real yield, higher vol
- $B(t, T) = E_t \left[ \frac{\xi(T)}{\xi(t)} \right]$ impacted by inflation expectations!

Question: How can we justify this in a consumption - based equilibrium model?
- How can differences in beliefs about inflation impact the value of a real cash flow when consumption is exogenous?
- Obvious response: consumption and inflation are correlated.
- This paper has a different answer
Model

- Output and price level

\[
\frac{d\epsilon(t)}{\epsilon(t)} = \mu_\epsilon dt + \sigma_\epsilon dz_\epsilon(t),
\]
\[
d\pi(t) = \pi(t)x(t)dt + \sigma_{\pi,\epsilon} dz_\epsilon(t) + \sigma_{\pi,USD} dz_{USD}(t)
\]

- \(x\) is mean reverting: differences in beliefs about rate of mean reversion and long run mean

- Preferences

\[
e^{-\rho t} \frac{1}{1-\gamma} \left( \frac{c_k(t)}{x(t)} \right)^{1-\gamma}
\]

- Complete markets: FOC of rep agent

\[
\lambda_{1,0} m_1(t) e^{-\rho t} c_1(t)^{-\gamma} x(t)^{(\gamma-1)} = \lambda_{2,0} m_2(t) e^{-\rho t} c_2(t)^{-\gamma} x(t)^{(\gamma-1)}
\]

- \(m_k(t) = \frac{dP_i(s)}{dP(s)}\), probability ratio (Radon - Nikodym derivative), exponential martingale

- \(\gamma\) plays double role
  - relative risk aversion: aversion to changes in consumption across states
  - reciprocal of EIS: aversion to changes in consumption over time
Theoretical result: SDF and Differences in beliefs

- FOC → consumption sharing rule

\[ \lambda_{1,0} m_{1,t} e^{-\rho t} c_1(t)^{-\gamma} x(t)^{-(\gamma-1)} = \lambda_{2,0} m_{2,t} e^{-\rho t} c_2(t)^{-\gamma} x(t)^{-(\gamma-1)} \]

\[ c_1(t) = \frac{m_{1(t)}^\frac{1}{\gamma}}{m_{1(t)}^\frac{1}{\gamma} + m_2(t)^\frac{1}{\gamma}} \epsilon(t) \]

- consumption sharing rule → SDF

\[ \xi(t) = m_k(t) c_k(t)^{-\gamma} x(t)^{-(\gamma-1)} \]

\[ = \left( \frac{\epsilon(t)}{x(t)} \right)^{-(\gamma-1)} \epsilon(t)^{-1} \left( m_{1(t)}^\frac{1}{\gamma} + m_{2(t)}^\frac{1}{\gamma} \right)^\gamma \]

\[ \text{diff. in beliefs abt inflation} \]

- \( \gamma = 1 \) (EIS = 1, sub. and income effects cancel), \( B(t, T) \) not impacted by consumption risk sharing, differences in beliefs don’t matter (Yan & Xiong, 2010)

- \( \gamma > 1 \) (EIS < 1, income effect dominates), \( B(t, T) \) impacted by consumption risk sharing, (this paper)
Mechanics

- price of real bond

\[ B(t, T) = E_t \left[ \left( \frac{\epsilon(T)}{x(T)} \right)^{-(\gamma - 1)} \left( \frac{\epsilon(t)}{x(t)} \right)^{(\gamma - 1)} \frac{\epsilon(T)^{-1}}{\epsilon(t)^{-1}} \left( \frac{m_1(T)^{\frac{1}{\gamma}} + m_2(T)^{\frac{1}{\gamma}}}{m_1(t)^{\frac{1}{\gamma}} + m_2(t)^{\frac{1}{\gamma}}} \right)^{\gamma} \right] \]

- Inflation shocks independent of output shocks

\[ B(t, T) = E_t \left[ \left( \frac{\epsilon(T)}{x(T)} \right)^{-(\gamma - 1)} \left( \frac{\epsilon(t)}{x(t)} \right)^{(\gamma - 1)} \frac{\epsilon(T)^{-1}}{\epsilon(t)^{-1}} \right] E_t \left[ \left( \frac{m_1(T)^{\frac{1}{\gamma}} + m_2(T)^{\frac{1}{\gamma}}}{m_1(t)^{\frac{1}{\gamma}} + m_2(t)^{\frac{1}{\gamma}}} \right)^{\gamma} \right] \]

\[ \text{indep. of diff. in beliefs} \]

\[ \text{dep. on diff. in beliefs} \]

- \( \gamma = 1 \) (Yan & Xiong, 2010), income and sub. effects cancel

\[ E_t \left[ \left( \frac{m_1(T)^{\frac{1}{\gamma}} + m_2(T)^{\frac{1}{\gamma}}}{m_1(t)^{\frac{1}{\gamma}} + m_2(t)^{\frac{1}{\gamma}}} \right)^{\gamma} \right] = E_t \left[ \left( \frac{m_1(T) + m_2(T)}{m_1(t) + m_2(t)} \right)^{\gamma} \right] = 1 \]

\[ \text{sum of 2 martingales} \]

- \( \gamma > 1 \) (this paper), income effect dominates
What makes inflation matter?

1. Traditional nominal rigidities in macro
   - Sticky prices and sticky wages

2. New nominal rigidities in macro
   - Sticky corporate leverage [Bhamra, Fisher & Kuehn (2011), Gomes, Jermann & Schmid (2013)]

How important is disagreement about inflation for macro-finance (via risk sharing channel)?
Inflation indexed bonds

A way to test whether disagreement about inflation really matters.

- In the model: inflation indexed bonds are a perfect hedge against inflation if you hold them to maturity, but earlier prices are not independent of inflation.
- Compute $\frac{\partial B(t,T)}{\partial \pi(t)}$ and $\frac{\partial B(t,T)}{\partial x(t)}$ to see how large they are.
- Do we see same signs and magnitudes empirically?
Recap: this paper

- Lucas endowment economy
- Agents with CRRA+multiplicative external habit and different beliefs about inflation
- Asset-Pricing Application: term structure of interest rates

There is more to asset pricing than term structure of bonds. There is more to household heterogeneity than differences in beliefs.
Asset-Pricing

1. Real risk-free and Nominal risk-free Bonds
   - Term structure
   - Vol, Inflation effects

2. Corporate Bonds
   - Credit Spread Puzzle, Leverage Puzzle,
   - Vol, Term structure
   - Cross-section
   - Inflation effects

3. Equities
   - Equity Risk Premium Spread Puzzle, Term structure
   - Sharpe Ratio Puzzle, Term Structure
   - Vol, Term structure
   - Predicability
   - Cross-section
   - Inflation effects

4. FX
   - UIP, Vol, ...

We would like one model which explains everything!
How well does the model in this paper do for equity return vol?
Equity return vol

- Disagreement about expected consumption growth and CRRA preferences: need $\gamma < 1$ ($EIS > 1$) to get equity return vol above dividend growth vol
- Is the same true with disagreement about inflation? Probably.
- With CRRA and differences in beliefs, cannot simultaneously match bond and equity market facts.
- Need to separate RRA and EIS (e.g. EZW), introduce heterogeneity in RRA, or add habits

This paper already has habits

- easy to simultaneously match equity return vol. and equity risk premium in addition to term structures of real and nominal bonds?
- What about time series predictability?
Equities: time series predictability

- Model: disagreement about expected endowment growth rate, no learning
- Look at correlation between the log price-dividend ratio, $\ln \frac{P^Y_t}{Y_t}$, and the realized excess return on the market $j$ years later, $R^Y_{t+j}$, for $j \in \{1, 2, 3, 5, 7\}$.
- All returns are annualized, model-based results are estimated on the basis of 50,000 years of simulated data.

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<th>3</th>
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Disagreement about expected endowment growth rate irrelevant for time series predictability of stock returns.
Disagreement about expected endowment growth rate irrelevant for time series predictability of stock returns.

Need to look at other forms of heterogeneity. [Chan & Kogan (2002)]
Conclusion

- In data: real yields impacted by differences in beliefs about inflation
- Nice theoretical result: real yields impacted by differences in beliefs about inflation even when inflation is independent of output. Driven by risk sharing across agents with different beliefs.
- How does disagreement about inflation compare with nominal rigidities?
- Can we do more than just look at risk-free bonds?
- Explore other forms of heterogeneity to match a wider set of asset pricing facts.