Network Centrality and the Cross Section of Stock Returns
by Kenneth Ahern

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Outline

- Aim
- Why do we care?
- Suggestions
Paper’s aim:

- Study how network effects impact stock returns across firms
- Do more central, i.e. ‘hub’ industries earn higher stock returns?
  - Fords chief executive, Alan R. Mulally, said the prospect of a failure of G.M. would cascade through the entire domestic auto industry and put millions of jobs at risk. [NYT, December 2, 2008]
Rephrasing the paper’s aims with basic theory

- No arbitrage & complete markets ⇒ ∃! strictly positive SDF, \( M \)
- Standard asset pricing equation

\[ P_{i,t} = E_t[M_{t,t+1}(D_{i,t+t} + P_{i,t+t})] \]  

(1)

- We are interested in returns: \( R_{i,t+1} = \frac{D_{i,t+t} + P_{i,t+t}}{P_{i,t}} \)

\[ E_t[R_{i,t+1}] - R_{f,t,t+1} = -R_{f,t,t+1}\text{Cov}_t[M_{t,t+1}R_{i,t+1}] \]

(2)

\[ = -R_{f,t,t+1}\rho_t[M_{t,t+1}, R_{i,t+1}]\text{Var}_t[M_{t,t+1}]\text{Var}_t[R_{i,t+1}] \]

(3)

- \( M_{t,t+1} \) depends on aggregate variables
- Kenneth’s question: does the centrality of a sector impact the covariance of its returns with the SDF?
Does centrality matter? Kenneth’s Answer

- Yes, centrality does matter.
- Industries more central in terms of the US inter-firm trade network have significantly higher stock returns than less centrally located industries
- Not related to previous features driving cross-sectional stock return variation (size, value, growth)
### Centrality

<table>
<thead>
<tr>
<th></th>
<th>Low 1</th>
<th>Low 2</th>
<th>Low 3</th>
<th>Low 4</th>
<th>High 5</th>
<th>1-5</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Monthly Returns (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levered</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value weighted</td>
<td>1.21</td>
<td>1.93</td>
<td>2.25</td>
<td>2.29</td>
<td>2.44</td>
<td></td>
<td>−1.23** (−2.24)</td>
</tr>
<tr>
<td>Firm-level equal weighted</td>
<td>0.28</td>
<td>0.97</td>
<td>1.30</td>
<td>1.23</td>
<td>1.42</td>
<td></td>
<td>−1.14*** (−4.69)</td>
</tr>
<tr>
<td>Industry-level equal weighted</td>
<td>0.10</td>
<td>0.68</td>
<td>0.95</td>
<td>0.85</td>
<td>1.22</td>
<td></td>
<td>−1.12*** (−4.14)</td>
</tr>
<tr>
<td>Unlevered</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value weighted</td>
<td>1.09</td>
<td>1.72</td>
<td>2.10</td>
<td>2.11</td>
<td>2.07</td>
<td></td>
<td>−0.99* (−1.84)</td>
</tr>
<tr>
<td>Firm-level equal weighted</td>
<td>0.29</td>
<td>0.91</td>
<td>1.19</td>
<td>1.15</td>
<td>1.29</td>
<td></td>
<td>−1.00*** (−4.33)</td>
</tr>
<tr>
<td>Industry-level equal weighted</td>
<td>0.16</td>
<td>0.66</td>
<td>0.84</td>
<td>0.74</td>
<td>1.04</td>
<td></td>
<td>−0.88*** (−3.56)</td>
</tr>
</tbody>
</table>

| Number of Industries | 59  | 77  | 82  | 83  | 84    |

### Panel B: Industry Characteristics

<table>
<thead>
<tr>
<th></th>
<th>Low 1</th>
<th>Low 2</th>
<th>Low 3</th>
<th>Low 4</th>
<th>High 5</th>
<th>1-5</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centrality</td>
<td>0.03</td>
<td>0.06</td>
<td>0.11</td>
<td>0.23</td>
<td>1.44</td>
<td></td>
<td>−1.41*** (−6.97)</td>
</tr>
<tr>
<td>Concentration of Customers</td>
<td>0.74</td>
<td>0.72</td>
<td>0.72</td>
<td>0.67</td>
<td>0.67</td>
<td></td>
<td>0.08* (1.95)</td>
</tr>
<tr>
<td>Concentration of Suppliers</td>
<td>0.61</td>
<td>0.58</td>
<td>0.60</td>
<td>0.62</td>
<td>0.68</td>
<td></td>
<td>−0.07*** (−4.68)</td>
</tr>
<tr>
<td>Log(Industry Output)</td>
<td>7.74</td>
<td>8.53</td>
<td>9.11</td>
<td>9.84</td>
<td>11.27</td>
<td></td>
<td>−3.53*** (−34.67)</td>
</tr>
<tr>
<td>Log(Industry Average Market Equity)</td>
<td>12.33</td>
<td>12.65</td>
<td>13.05</td>
<td>13.25</td>
<td>13.82</td>
<td></td>
<td>−1.48*** (−6.41)</td>
</tr>
<tr>
<td>Log(Industry Median Market Equity)</td>
<td>12.15</td>
<td>12.34</td>
<td>12.51</td>
<td>12.46</td>
<td>12.57</td>
<td></td>
<td>−0.42* (−1.89)</td>
</tr>
</tbody>
</table>
Reconnecting with basic theory

\[ E_t[R_{i,t+1}] - R_{f,t,t+1} = -R_{f,t,t+1} \rho_t[M_{t,t+1}, R_{i,t+1}] \text{Var}_t[M_{t,t+1}] \text{Var}_t[R_{i,t+1}] \]

- Cross-sectional variation in \( E_t[R_{i,t+1}] - R_{f,t,t+1} \) driven by cross-sectional variation in \( \rho_t[M_{t,t+1}, R_{i,t+1}] \) or \( \text{Var}_t[R_{i,t+1}] \).
- From the point of view of network effects and centrality, \( \rho_t[M_{t,t+1}, R_{i,t+1}] \) should be the important part.
Why do we care?

- Cross-sectional asset pricing focuses on relating differences in stock returns to characteristics, e.g. size/value/growth
- Cross-sectional puzzles in asset pricing should ultimately be related to sector/industry/firm specific characteristics.
  - Ken’s work can be seen as an example of this, focusing on the economically intuitive idea of industry centrality within a network
- Eventually use understanding of cross-sectional risk premia to assess policy implications on a sector by sector basis.
  - The US car industry was bailed out, because of its perceived centrality
  - Would an increase in expected stock returns for the car industry increase expected aggregate stock returns?
  - What about the welfare implications?
Suggestions

- More data (network matrix based on 1 year of data). Obvious point.
- Explain centrality measure more clearly. Key concept: I did not get much intuition from the paper.
- Directly test a theory stated mathematically rather than a qualitative hypothesis.
  - Existing theory provides conditions under which idiosyncratic shocks can have an aggregate impact [Acemoglu, Carvalho, Ozdaglar, and Alireza Tahbaz-Salehi, (2012), The network origins of aggregate fluctuations, Econometrica] Are these conditions satisfied empirically?
- Risk premia are driven by network changes
Think about economy with one sector that provides inputs to itself and all the other sectors.

Sector 1 is very central.

Write network as matrix \((n=4)\).

\[
\begin{bmatrix}
  x_{11} & 0 & 0 & 0 \\
  x_{21} & 0 & 0 & 0 \\
  x_{31} & 0 & 0 & 0 \\
  x_{41} & 0 & 0 & 0 \\
\end{bmatrix}
\]

Sector \(i\) receives \(x_{ij}\) from Sector \(j\)

Normalize matrix so elements of columns sum to 1. Get a new matrix \(W=(w_{ij})\), where

\[
w_{ij} = \frac{x_{ij}}{\sum_{i=1}^{4} x_{ij}}
\]

\[
\begin{bmatrix}
w_{11} & 0 & 0 & 0 \\
w_{21} & 0 & 0 & 0 \\
w_{31} & 0 & 0 & 0 \\
w_{41} & 0 & 0 & 0 \\
\end{bmatrix}
\]
What can we do with $W$?

We can see how a vector of inputs $x = (x_1, \ldots, x_n)^T$ is distributed across the 4 sectors via $Wx$, e.g.

$$
\begin{pmatrix}
  w_{11} & 0 & 0 & 0 \\
  w_{21} & 0 & 0 & 0 \\
  w_{31} & 0 & 0 & 0 \\
  w_{41} & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
  1 \\
  0 \\
  0 \\
  0
\end{pmatrix}
= (w_{11}, w_{21}, w_{31}, w_{41})^T
$$

An input produced by sector 1 gets transferred to all 4 sectors.

What about input produced by sector 2?

$$
\begin{pmatrix}
  w_{11} & 0 & 0 & 0 \\
  w_{21} & 0 & 0 & 0 \\
  w_{31} & 0 & 0 & 0 \\
  w_{41} & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
  0 \\
  1 \\
  0 \\
  0
\end{pmatrix}
= (0, 0, 0, 0)^T
$$

It does not go anywhere. Sector 2 is at the edge of the trade network.
After many rounds of intersectoral trade a vector of inputs $x$ will get mapped into a new vector.

After $k$ rounds of trade

$$W^kx = \text{new input vector}$$

If Sector $j$ is more central, it will have more input passing through it: the $j$’th element of the new input vector will be larger.

So what has $W$ got to do with which sectors are more central than others?
Where do the eigenvectors of $W$ tie in?
Use eigenvectors to compute the new vector $W^kx$
Centrality III

- $W$ has eigenvectors $e_i, \ i \in \{1, \ldots, 4\}$ with corresponding eigenvalues $\lambda_i, \ i \in \{1, \ldots, 4\}$
  \[ We_i = \lambda_i e_i \] (5)

- Any vector of inputs $x$ can be written wrt to the basis of eigenvectors of $W$ as
  \[ x = \sum_{i=1}^{4} \text{scalar} b_i \times \text{eigenvector } e_i \] (6)

- After one round of intersectoral trade, input is transferred as follows
  \[ Wx = \sum_{i=1}^{4} \lambda_i b_i e_i \]

- After $k$ rounds of intersectoral trade, initial input is transferred as follows
  \[ W^k x = \sum_{i=1}^{4} b_n \lambda_n^k e_n \]

- Most of the eigenvalues make negligible contribution to the final vector. What matters is the contribution from largest (dominant) eigenvalue ($i = d$)
  \[ W^k x \approx b_d \lambda_d^k e_d = b_d \lambda_d^k (e_{d,1}, \ldots, e_{d,4})^T \] [Perron-Frobenius Theory]

- $e_{d,i}$ is the $i$’th element of principal eigenvector of $W$: measures the centrality of Sector $i$
Existing theory: adding up risks

Acemoglu et al (micro foundations for macro risk):

\[
\text{Var} \left[ \sum_{i=1}^{n} v_i \tilde{\epsilon}_i \right] \sim \left( \frac{1}{\sqrt{n}} + \text{dominant sector effect + interconnectivity effect} \right)
\]  

- dominant sector effect: only a small fraction of sectors are responsible for the majority of the input supplies in the economy.
  - shocks to dominant sectors propagate through the entire economy as their low productivity leads to lower production for all of their downstream sectors

- interconnectivity effect
  - A group of sectors has common suppliers. A shock to the the suppliers propagates through the sectors connected to them.

Relate the dominant sector effect and the connectivity effect to the properties of the trade matrix.

- Lots of work on this: algebraic graph theory applied to weighted digraphs

Are this effects quantitatively important? Can you distinguish between them?
(Pseudo) theory

Variance is not enough. Risk premia are driven by covariances

\[
E_t[R_{i,t+1}] - R_{f,t,t+1} = -R_{f,t,t+1} \rho_t[M_{t,t+1}, R_{i,t+1}] \text{Var}_t[M_{t,t+1}] \text{Var}_t[R_{i,t+1}]
\] (8)

Special case: Extension of CAPM

\[
M_{t,t+1} = A + F_1 R_{W_{t+1}} + \sum_{j=1}^{K} F_j Z_{j,t+1},
\] (9)

where

\[
R_{W,t+1} = \sum_{i=1}^{I} \frac{P_{i,t}}{W_t} R_{i,t+1}
\] (10)

\[Z_{j,t+1}\] other factors

Compute \(\text{Cov}_t[\sum_{i=1}^{I} \frac{P_{i,t}}{W_t} R_{i,t+1}, R_{i,t+1}]\) using the algebra in Acemoglu et al.

- Identify the extra terms which arise due to network effects. How large are they?
- Do the same for other factors, in particular sector size.
Final comment

- Asset pricing theory links conditional expected returns to second moments of changes (in prices, dividends, consumption, wealth, wealth distribution).
- Data on centrality focuses on a snapshot of a input/output matrix.
- The risk of a change in network structure is what is priced; a static network where there is no chance of sectors gaining/losing centrality will not generate risk premia.
- What we really need to know is how the principal eigenvector of the input/output matrix changes.
  - The more stable principal eigenvector is wrt network changes, the smaller risk premia will be.
- Theorists: go and study how eigenvectors change.
- Empiricists: go and study how to measure eigenvector changes.
Conclusion

- Very exciting and potentially very useful research topic.
- Hopefully Kenneth’s centrality will be very high in the near future.