Volatility, the Macroeconomy and Asset Prices
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My view of paper’s aim:

Study how volatility in risk prices combined with volatility in returns on aggregate consumption claim impact
- joint dynamics of risk-return relation for human capital and aggregate equity
- cross-section of equity risk premia
Why do we care?

- Why do we care about stochastic risk prices?
- Why do we care about valuing human capital and aggregate equity?
- Why do we care about cross-sectional asset pricing?
Value of human capital has large impact on welfare.
- Share of human wealth in overall wealth $\approx 80\%$, (Lustig & Van Niewerburgh, 2008)
- All progress ultimately depends on human ingenuity: correct valuation of human capital has important implications for investment in education, etc.

To correctly assess welfare implications of policies need a macro-finance model which accurately values human capital in addition to aggregate cash flows.

Cross-sectional puzzles in asset pricing should ultimately be related to sector/industry/firm specific characteristics. Eventually use understanding of cross-sectional risk premia to assess policy implications on a sector by sector basis.
Stochastic risk prices. Why?

- Aggregate risk premium important because large aggregate risk premium $\iff$ large welfare cost of business cycles (Lucas)
- Risk premia are determined by risk prices
  - No arbitrage & complete markets $\Rightarrow \exists$! strictly positive SDF, $M = e^m$
  - Standard dynamic asset pricing eqn:
    $$E_t[r_{t+1} - r_f] = -\text{Cov}_t(m_{t+1} - E_t[m_{t+1}], r_{t+1} - E_t[r_{t+1}])$$
    unexp. change in log SDF: gives risk prices
- Facts about risk premia
  - Market risk premium are large relative to 1st generation models (Mehra –Prescott)
  - Market risk premium is stochastic (time – varying) (Schiller) – countercyclical
- Implications for risk prices
  - Risk prices are large and stochastic (countercyclical)
Finding a sensible SDF

- Minimum requirements: SDF with large and stochastic (countercyclical) risk price
- Adding economic meat to financial ketchup: get SDF from assumptions about household preferences and aggregate consumption
- 1st generation models: CRRA + log normal consumption – risk premia small and constant: price of risk small and constant
- Choose one of several 2nd generation models which generate reasonable aggregate risk premium with large and countercyclical risk prices
  - Campbell-Cochrane (external habit)
  - Bansal & Yaron (LRR)
  - time-varying disaster risk (Rietz, Barro)
- Maybe explore a 3rd generation model?:
  - CRRA, heterogeneity in risk aversion, OLG
  - CRRA, heterogeneity in beliefs and learning, OLG
- Beware: all above models are consumption based. None include labor income in dynamic budget constraint or labor/leisure in utility function. Limits understanding of how human capital is valued.
This paper: LRR SDF

- Source of large and stochastic (countercyclical) risk price
  - Continuum of identical EZW households: consumers, no labor/leisure trade-off. Get single EZW rep agent who consumes aggregate cons.
  - Exog. agg. cons: conditionally lognormal with stochastic expected cons growth, which itself has stochastic vol

\[
\begin{align*}
    c_{t+1} - c_t &= \mu + x_t + \sigma \eta_{t+1} \\
    x_{t+1} &= \rho x_t + \phi e \sigma_t \epsilon_{t+1} \\
    \sigma^2_{t+1} &= \sigma^2_c + \nu \left( \sigma^2_t - \sigma^2_c \right) + \sigma_w \omega_{t+1}
\end{align*}
\]

- \( \gamma \neq \frac{1}{\psi} \): rep. agent cares about whether uncertainty is resolved sooner or later
  - shocks to expectations are priced
  - vol of shocks to expectations are priced
\( m_{t+1} - E_t[m_{t+1}] = -\gamma \eta_{t+1} \)

price of shock to expected cons growth: this is stochastic

\[- \left( \gamma - \frac{1}{\psi} \right) \frac{\kappa_1}{1 - \kappa_1 \rho} \phi e^{\sigma t} \epsilon_{t+1} \]

\[- \left\{ - \left( \gamma - \frac{1}{\psi} \right) \left( \gamma - 1 \right) \frac{\kappa_1}{1 - \nu \kappa_1} \frac{1}{2} \left[ 1 + \left( \frac{\kappa_1}{1 - \rho \kappa_1} \right)^2 \right] \sigma_w \right\} w_{t+1} \]

price of vol of shock to expected cons. growth

- Preference for early resolution of intertemporal risk: \( \gamma > \frac{1}{\psi} \)
- Increases size of risk price
- Combined with stoch. vol., \( \sigma_t \): countercyclical price of expected cons. growth risk
Defining volatility

\[ V_t = \frac{1}{2} \text{Var}_t[m_{t+1} + r_{c,t+1}] \]  \hspace{1cm} (1)
Human capital and news about expected consumption growth

- **LRR SDF ⇒**

\[
(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1 (c_{t+j+1} - c_{t+j}) = \psi N_{DR,t+1} - \frac{\psi - 1}{\gamma - 1} N_{V,t+1}
\]

where

\[
N_{DR,t+1} = (E_{t+1} - E_t) \left( \sum_{j=1}^{\infty} \kappa_1^j r_{c,t+1} \right)
\]

\[
N_{V,t+1} = (E_{t+1} - E_t) \left( \sum_{j=1}^{\infty} \kappa_1^j V_{t+j} \right)
\]

- Assume share of human wealth on total wealth is constant, \( \omega \)

\[
N_{DR,t+1} = \omega N_{DR,t+1}^Y + (1 - \omega) N_{DR,t+1}^d
\]
Main eqn for understanding news about human capital discount rate

\[
(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1 (c_{t+j+1} - c_{t+j}) = \psi N_{DR,t+1} - \frac{\psi - 1}{\gamma - 1} N_{V,t+1}
\]

\[
N_{DR,t+1} = \omega N_{DR,t+1}^y + (1 - \omega) N_{DR,t+1}^d
\]

Without stochastic risk price, \( N_{DR,t+1}^d \) and \( N_{DR,t+1}^y \) must be of opposite sign to make sure news about cons growth is small enough: returns on human capital and dividends negative correlated (Lustig & Van Niewerburgh, 2008)

With stochastic risk price and \( \gamma > 1, \psi > 1 \) (implied by \( \gamma > \frac{1}{\psi} > 1 \)), can have \( N_{DR,t+1}^d \) and \( N_{DR,t+1}^y \) of the same sign: returns on human capital and dividends positively correlated
Importance of labor income

1. Labor income is part of dynamic budget constraint
2. Could (should?) put labor into utility function: figure out new SDF

- Just do 1 (quick and dirty).

\[ W_{t+1} = (W_t + E_t - C_t)R_{c,t+1} \]  (5)

\[ W_{t+1} = (W_t - C_t^-)R_{c,t+1} \]  (6)

- cons minus labor income

But this does not impact SDF, so still have

news about cons growth

\[
(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1(c_{t+j+1} - c_{t+j}) = \psi N_{DR,t+1} - \frac{\psi - 1}{\gamma - 1} N_{V,t+1}
\]

\[ N_{DR,t+1} = \omega N_{DR,t+1}^\gamma + (1 - \omega)N_{DR,t+1}^d \]
EZW preferences with labor

- Put labor into utility function: change SDF and get new expression for how news about consumption growth is related to news about vol shocks

\[ U_t = f(C_t^*, CEQ_t[U_{t+1}]), \]
\[ f(x, y) = \left( x^{1-\frac{1}{\psi}} + y^{1-\frac{1}{\psi}} \right)^\psi \]
\[ CEQ_t[U_{t+1}] = \left( E_t[U_{t+1}^{1-\gamma}] \right)^{1-\gamma} \]
\[ C_t^* = g(C_t, \bar{N} - N_t) \]
\[ = C_t(\bar{N} - N_t)^\tau \]

- E.g. Kung (2012)
New SDF with labor

- new log SDF

\[ m_{t+1} = \text{cst} - \frac{1}{\psi} \Delta c_{t+1} + \tau \left( 1 - \frac{1}{\psi} \right) \begin{pmatrix} \Delta l_{t+1} \\ \Delta w_{t+1} \end{pmatrix} + \text{cst} r_{c,t+1} \]

- Will we risk prices be such that we get a realistic equity premium?
- Risk-free rate?
- \( \psi > 1? \)
New eqn

\[
(E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1 (c_{t+j+1} - c_{t+j})
\]

\[
- c_{st} \cdot (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1 (l^*_{t+j+1} - l^*_{t+j})
\]

\[
= c_{st} \cdot N_{DR,t+1} - c_{st} \cdot N_{V,t+1}
\]

\[
N_{DR,t+1} = \omega N^y_{DR,t+1} + (1 - \omega) N^d_{DR,t+1}
\]

Sign of last cst crucial for vol risks
Volatility for aggregate stock returns > Vol in aggregate dividend growth

\[
r_{t+1} - E_t[r_{t+1}] = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1}
\]

\[
= ECF_{t+1} - EDR_{t+1}
\]

- expected DR news drives unexpected change in stock returns