Risk-Adjusted Capital Allocation and Misallocation
Discussion

Harjoat S. Bhamra

Imperial College Business School

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Aims

- A cross-sectional macro-finance paper

- Connect variations in the marginal productivity of capital ($MP_K$) to variations in expected risk premia
Why do we care?

- Cross-sectional puzzles in asset pricing should ultimately be related to sector/industry/firm specific characteristics.

- Eventually use understanding of cross-sectional risk premia to assess policy implications on a sector by sector basis.
Outline of Paper

- Build partial equilibrium model of firms

  \( PV \) of future \( MPK \) equal across firms. Don’t ignore differences between \( \mathbb{P} \) and \( \mathbb{Q} \)

- Firm-level operating profits are decreasing with the stochastic price of risk – extent of this loading is different across firms.

- Generates heterogeneity in both \( MPK \) and expected risk premia.

- Use model to infer heterogeneity in \( MPK \) from heterogeneity in risk premia.

- Compare inferred heterogeneity in \( MPK \) with direct measure – how close are they?
Model

- Cross-section of firms. Firm $i$ pays out dividend flow $D_{i,t}$

\begin{equation}
D_{i,t} + l_{i,t} = \Pi_{i,t}
\end{equation}

- Bellman equation

\begin{equation}
V_{i,t} = \sup_{K_{i,t+1}} D_{i,t} + E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} V_{i,t+1} \right]
\end{equation}

where

\begin{equation}
K_{i,t+1} = l_{i,t} + (1 - \delta) K_{i,t}
\end{equation}

- FOC

\begin{equation}
E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} [\Psi_{i,t+1} + (1 - \delta)] \right] = 1,
\end{equation}

where

\begin{equation}
\Psi_{i,t+1} = \frac{\partial \Pi_{i,t+1}}{\partial K_{i,t+1}}
\end{equation}
Interpreting the FOC

- Present value of future marginal product of capital is constant across firms

\[
E_t \left[ \frac{\Lambda_{t+1}}{\Lambda_t} \psi_{i,t+1} \right] \text{ is independent of } i
\]

where

\[
\psi_{i,t+1} = \frac{\partial \Pi_{i,t+1}}{\partial K_{i,t+1}}
\]

- Explicitly change measure from \( P \) to \( Q \)

\[
E^Q_t \left[ e^{-rf,t} \psi_{i,t+1} \right] \text{ is independent of } i \Rightarrow E^Q_t \left[ \psi_{i,t+1} \right] \text{ is independent of } i
\]

- Risk-neutral expected value of future marginal product of capital is identical across firms

- Interpret in two ways
  1. Old fashioned macro view
  2. Modern macro-finance view
Old fashioned macro view

- \( \mathbb{P} \) and \( \mathbb{Q} \) are same (risk premia are just noise) and so

\[
E_t^Q [\Psi_{i,t+1}] \text{ is independent of } i \Rightarrow E_t [\Psi_{i,t+1}] \text{ is independent of } i
\]

- Then look at data and see that \( E_t [\Psi_{i,t+1}] \) is not independent of \( i \) and deduce that there is misallocation

- Explore welfare implications of this misallocation.
Modern macro-finance view

- $\mathbb{P}$ and $\mathbb{Q}$ are not same and so

\[(10)\]
\[E_t^Q [\Psi_{i,t+1}] \text{ is independent of } i \text{ but } E_t [\Psi_{i,t+1}] \text{ can depend on } i\]

- Then look at data and see that $E_t [\Psi_{i,t+1}]$ is not independent of $i$ and deduce that there are risk premia instead of misallocation.
  - At the very least – misallocation must be less than the traditional macro view suggests.

- Empirical asset pricing – subset focuses on understanding cross-sectional differences in risk premia.

- Why not connect cross-sectional differences in $E_t [\Psi_{i,t+1}]$ to cross-sectional differences in risk premia?
Need structure on $\Pi_{i,t}$ and SDF $\Lambda$ to exploit FOC

Assume $\pi_{i,t} = \ln \Pi_{i,t}$ and SDF $\lambda_t = \ln \Lambda_t$ are linear functions of Gaussian rv’s.

\[
E_Q^t [e^{-rf,t}[\Psi_{i,t+1} + (1 - \delta)]] = 1 \Rightarrow \ln E_Q^t [\Psi_{i,t+1}] \approx r_{f,t} + \delta
\]

\[
E_Q^t [\psi_{i,t+1}] \approx r_{f,t} + \delta - \frac{1}{2} \text{Var}_t^Q [\psi_{i,t+1}]
\]

Use above equation to derive $k_{i,t+1}$ – will depend on risk-neutral expectations

Marginal product of capital will depend on risk-neutral expectations
Assumptions on operating profits and SDF

(13) \[ \Pi_{i,t} = Ge^{\beta_i x_t + z_{i,t} + \theta k_{i,t}} \Rightarrow \psi_{i,t} = g + \beta_i x_t + z_{i,t} - (1 - \theta)k_{i,t} \]

(14) \[ \lambda_{t+1} - \lambda_t = E_t[\lambda_{t+1} - \lambda_t] - (\gamma_0 + \gamma_1 x_t)\sigma_\lambda \epsilon_{t+1} \]

where

(15) \[ x_{t+1} = \rho x_t + \sigma_x \epsilon_{t+1} \]

What does this mean?

- \((\gamma_0 + \gamma_1 x_t)\sigma_\lambda\) is the price of risk, \(\gamma_1 < 0\) – low \(x\) means higher price of risk

- \(\epsilon\) is part of what describes the aggregate state – unexpected improvement in \(\epsilon\) leads to negative shock to SDF and increase in \(x\)

- Operating profits load on the price of risk and not on \(\epsilon\)
FOC leads to

\[ k_{i,t+1} = g + \beta_i \frac{E_t^Q[x_{t+1}]}{1 - \theta} + E_t[z_{i,t+1}] - (r_{f,t} + \delta) \]

Plausible that have high price of risk (low \( x \)) in bad aggregate states.

\[ E_t^Q[x_{t+1}] = \rho x_t - (\gamma_0 + \gamma_1 x_t)\sigma_x \sigma_x = \hat{\rho} x_t - \hat{\gamma}_0. \]

\[ k_{i,t+1} \propto \beta_i \frac{\hat{\rho} x_t}{1 - \theta} + E_t[z_{i,t+1}] \]

\( k_{i,t+1} \) lower when risk is priced more severely – real investment lower in bad aggregate states

if real interest rate is lower in bad states (higher precautionary savings demand in bad states or basic intertemporal smoothing) this effect is reduced – good because real investment less volatile than price of risk
log marginal product of capital

(18) \[ \psi_{i,t} \propto \left( 1 + \hat{\rho} \frac{\theta}{1 - \theta} \right) \beta_i x_t \]

(19) \[ E_t[\psi_{i,t+1}] \propto \left( 1 + \hat{\rho} \frac{\theta}{1 - \theta} \right) \beta_i \rho x_t \]

Expected risk premium of firm \( i \)

(20) \[ \ln E_t[R_{i,t+1}^e] \propto \beta_i (\gamma_0 + \gamma_1 x_t) \]

Model connects expected risk premia to cross-section of marginal products of capital.
Cross-sectional variance of expected risk premia v. marginal products of capital

- Cross-sectional variance of expected risk premia is just under half of the cross-sectional variance of marginal products of capital.

- The modern macro-finance view suggests there is less misallocation than the traditional macro view.

- Heterogeneity in $\delta$, $\theta$ does not generate in enough variance in expected risk premia.
Summary of Contributions

- Differences in exposure of operating profits to the price of risk can explain both cross-sectional variation in expected risk premia and MPK.

- Differences in firm-level production parameters can explain some of the cross-sectional variation in MPK, but not expected risk premia.
Comments I

- Do operating profits really depend on the price of risk?
- Could you get this in general equilibrium – doubtful.
Consider a model with growth options – very common in cross-sectional asset pricing literature.

Rise in fundamental volatility makes growth options more valuable – investment is delayed – will impact MPK. Will also impact expected risk premium.

There is cross-sectional heterogeneity in the proportion of firm value derived from growth options versus assets in place.

Heterogeneity in growth options/assets in place will generate differences in expected risk premia and MPK in a model with stochastic volatility.

Try and build a ge model starting from Gomes, Kogan & Zhang (2003)?
In a model with no frictions, there is no misallocation.

Does it really make sense to talk about reductions in TFP stemming from misallocation, when heterogeneity in MPK is an efficient outcome?

If it’s not misallocation, why call it misallocation?