Do Short-selling Constraints Matter?
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Paper’s aim:

- How do short selling constraints impact the relationship between prices and fundamentals
  - Focus on long-run prices

\[
\lim_{t \to \infty} P_t \quad (1)
\]

Key features

- Standard Diamond & Verrecchia (1987) framework
- One important change: make proportion of informed traders unknown, but with a known distribution.
Why do we care?

Short sales constraints and you

- Regulators restrict short selling to decrease potential downward price cascades. Counterargument: this only contributes to market inefficiency.
- Would like to understand impact of short selling constraints on
  - Financial variables
    - prices, returns, risk premia, volatility, correlation, etc
  - Real variables
    - real investment
    - production
    - household welfare ← YOU
- If anything happens, starts with financial variables and can feed into real economy, affecting welfare
- Diamond & Verrecchia (1987)
  - \( \lim_{t \to \infty} P_t \) not impacted by short sales constraints
  - should be no long-run impact on welfare
- This paper
  - \( \lim_{t \to \infty} P_t \) is impacted by large short sales constraints
  - should be a long-run impact on welfare
Model Summary I

- Two assets, ex-ante identical
- \( t \in \mathbb{N} \)
- **cash flow**, \( v \) (fundamental value) fixed over time, but which value does it take? Common knowledge: \( \Pr(v = 1) = \lambda, \Pr(v = 0) = 1 - \lambda \).
- risk-neutral **market makers**: no inventory costs or constraints
- continuum of risk-neutral **traders**
  - \( \mu \) are **Informed**
  - \( 1 - \mu \) are **Uninformed**: 1/2 buy and 1/2 sell in any period
  - fraction \( \rho \) own asset 1
  - fraction \( 1 - \rho \) own asset 2
  - \( \mu \) unknown constant, common knowledge: pdf \( f(\mu) \)
  - \( \rho \) known constant

- What do Informed traders know?
  - iid signal \( s_i \) for asset \( i \)
  - \( \Pr(s_i = 1|v_i = 1) = \Pr(s_i = 0|v_i = 0) = \phi \in (1/2, 1) \)
Informed traders use information in signals

Posterior belief via Bayes’ Thm

\[
\beta(v_i = 1 | s_i = 1) = \frac{\Pr(s_i = 1 | v_i = 1) \Pr(v_i = 1)}{\Pr(s_i = 1 | v_i = 1) \Pr(v_i = 1) + \Pr(s_i = 1 | v_i = 0) \Pr(v_i = 0)} = \frac{\phi \lambda}{\phi \lambda + (1 - \phi)(1 - \lambda)} > \lambda
\]
Market makers and prices.

- Market makers set price to maximize expected profits ⇒ in period 1

\[ A_1 = E[v | h^1 = +1] \]  \hspace{1cm} (2)

\[ B_1 = E[v | h^1 = -1] \]  \hspace{1cm} (3)

Prices impacted by order flow.
Long-run price with no short sale constraints

- What about price in the long-run as $t \to \infty$
  \[
  \lim_{t \to \infty} P_t = E[v|\text{infinite seq of buys and sells}]
  \] (4)

- In the long-run, what proportion of trades will be buys and sells?

- Conditional on $v = 1$, the fraction of buy orders for asset 1 is
  \[
  \frac{\Pr(buy|v = 1)}{\text{fraction of traders trading asset 1}} = \frac{\frac{1}{2}(1 - \mu)\rho + \mu\phi}{1 - (1 - \mu)(1 - \rho)}
  \] (5)

- Conditional on $v = 0$, the fraction of buy orders for asset 1 is
  \[
  \frac{\Pr(buy|v = 0)}{\text{fraction of traders trading asset 1}} = \frac{\frac{1}{2}(1 - \mu)\rho + \mu(1 - \phi)}{1 - (1 - \mu)(1 - \rho)}
  \] (6)

- Fraction of buy orders cond. on $v = 1 > \frac{1}{2} > \text{fraction of buy orders cond. on } v = 0$, $\forall \mu \in [0, 1]$

- After many observations, if fraction of buy orders $> \frac{1}{2}$, know $v = 1$ and if fraction of buy orders $< \frac{1}{2}$, know $v = 0$. **Through observing order flow eventually learn what $v$ is.** Therefore,
  \[
  \lim_{t \to \infty} P_t = v.
  \] (7)
Ban short sales!

Impacts the order flow: fraction of buys in long-run order flow sequence will be different.

Can find values of $\mu$ such that market makers no longer learn $v$ as $t \to \infty$
Summary

- uncertainty about $\mu$ (fraction of informed traders) + short sale ban $\Rightarrow$ cannot learn $v$ from order flow as $t \to \infty$
Real life short sale constraints

- Usually, only a small proportion of the market is affected.
- If we impose short sales constraints on a few stocks, there will be an eqm impact on other stocks, but will it really be of any consequence?
- Any welfare effects tiny?
Number of Assets

- Clarify why need 2 assets:
  - ownership effects? is that it?
  - I think one asset would be fine
Path Dependence

Look at predictability, momentum?
Price Stationarity

- assumptions about \( v \) being fixed over time, not realistic when thinking about long-run prices.
- prices may look stationary over 1 day, but not over 10 years

\[
V \in \{A + B \cdot t - 1, A + B \cdot t + 1\} \tag{8}
\]

- Perhaps make \( B \) unknown instead of \( \mu \)?
Framing the discussion of size of short selling costs

- Cost of short selling is huge: equals fundamental value.
- Would 1% of fundamental value be ok? Or is that too small?
page 9, Proof of Prop 3, 8th line from bottom, $p$, should be a $\rho$
I enjoyed working through the proofs.

Write another paper like this!

Explore results on path dependence more fully.