Arbitrage-Free Bond Pricing
With Dynamic Macroeconomic Models
Discussant: Harjoat S. Bhamra, UBC

Michael F. Gallmeyer       Burton Hollifield

Francisco Palomino       Stanley E. Zin

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The Bigger Picture

- There is interaction between
  1. Monetary Policy
  2. Real Economy
  3. Asset Prices

- Some research has focused on Monetary Policy and the Real Economy (Woodford, sticky price/wage models)
Asset Prices and Monetary Policy

• What about Asset Prices?
• When should Monetary Policy take Asset Prices into consideration?
  - Greenspan and asset price bubbles, Bernanke and the credit crisis
• If Monetary Policy does take Asset Prices into account, what are the real effects? What are the effects on Portfolio Choices and Asset Prices?
• Large research agenda: needs some focus!
What This Paper Does

- Makes inflation a consequence of monetary policy
- How is it done?
  1. Assumption about monetary policy
     - Nominal interest rate (nominal yield on 1 year bond) follows a Taylor Rule
     \[ i_t = -\ln b_t^{1,$} = f(\text{real cons growth, cons growth vol, } p_t, \text{Exog Shocks}) \]
  2. Introduce a real SDF (\( m_t \))
     - Find the price level which makes nominal interest rate consistent with no-arbitrage
- Real SDF is not exogenous as in standard term structure models
  - Real SDF comes from representative agent’s preferences and exogenous real consumption (Lucas)
Questions We Can Answer With This Paper’s Model

- Can ask how the following affect inflation and the nominal yield curve:
  - Changing $f$ (Taylor Rule)
  - Changing the rep agent’s preferences
  - Statistical assumptions about real consumption growth
Issues Beyond The Model

- Can households and a monetary policy setting agent be aggregated into a rep agent?
- Monetary policy has no real implications: real consumption is exogenous
  - Impossible to assess welfare implications of monetary policy (see Palomino (2007))
- The only way monetary policy affects nominal yields is via inflation
How Does Inflation Affect Nominal Bond Prices?

• Price of a nominal bond
  • Date-$t$ price of a bond which pays out one dollar at date $t+n$

$$b_{t}^{n,\$} = b_{t}^{n} E_t[P_t/P_{t+n}] \left[ 1 + \text{Cov}_t \left( \frac{m_t/m_{t+n}}{E_t[m_{t+n}/m_t]}, \frac{P_t/P_{t+n}}{E_t[P_t/P_{t+n}]} \right) \right]$$

• $p_t = \ln P_t$, $m_t = P_t m_\$$

• Expected inflation

• Covariance of inflation with real SDF- Inflation Risk Premium
The Model Framework: **Endogenous Inflation Model**

- **Taylor Rule**
  \[ i_t = -\ln b_t^1, \quad b_t^1 = f(x_t, \text{cons growth vol, } p_t, \text{Exog Shocks}) \]

  But
  \[ b_t^1 = b_t E_t[P_t/P_{t+1}] \left[ 1 + \text{Cov}_t \left( \frac{m_t/m_{t+1}}{E_t[m_{t+1}/m_t]}, \frac{P_t/P_{t+1}}{E_t[P_t/P_{t+1}]} \right) \right] \]

- Inflation is now endogenous!
Consumption Growth

- Consumption growth $x_{t+1} = c_{t+1}/c_t$ is mean-reverting with stochastic vol $\nu_t$

$$x_{t+1} = (1 - \phi_x)\theta_x + \phi_x x_t + \nu_t^{1/2}\epsilon_{t+1}^x$$

$$\nu_{t+1} = (1 - \phi_v)\theta_v + \phi_v \nu_t + \sigma_v \epsilon_{t+1}^v$$
Representative Agent

- Real SDF ($m$) comes from a representative agent with Epstein-Zin preferences
  - RRA, $\gamma$
  - EIS, $\psi$
- Agent cares about path of future consumption: continuation utility normalized by its CEQ enters the SDF

$$m_{t+1} = \beta x_{t+1}^{1/\psi} \left( \frac{U_{t+1}}{CEQ_t(U_t)} \right)^{1/\psi-\gamma}$$

$$u(CEQ_t(U_{t+1})) = E_t(u(U_{t+1})),$$

where

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}$$
Exogenous Monetary Policy & Endogenous Inflation

• Monetary policy is chosen exogenously (Taylor Rule)

\[ i_t = -\ln b_t^1 = \bar{\tau} + \tau_x x_t + \tau_p p_t + s_t \]

• Exogenous \( s_t \), AR(1), orthogonal to shocks driving \( x_t \) and \( v_t \)
• Obtain endogenous inflation

\[ p_t = \bar{\pi} + \pi_x x_t + \pi_v v_t + \pi_s s_t \]
What Does Endogenizing Inflation Buy Us?

- **Exogenous inflation**
  - The exogenous inflation process in the paper does not covary with real SDF → zero inflation risk premium → downward sloping nominal term structure
  \[
  b_t^n \Delta = b_t^n E_t[P_t/P_{t+n}]
  \]

- **Endogenous inflation**
  - Endogenous inflation process in the paper does covary with real SDF → non-zero inflation risk premium → upward sloping nominal term structure & more volatility at long end
  \[
  b_t^n \Delta = b_t^n E_t[P_t/P_{t+n}] \left[1 + \text{Cov}_t \left( \frac{m_t/m_{t+n}}{E_t[m_{t+n}/m_t]}, \frac{P_t/P_{t+n}}{E_t[P_t/P_{t+n}]} \right) \right]
  \]
Exogenous v Endogenous Inflation

- Could just set exogenous inflation equal to the endogenously derived inflation process and obtain identical nominal term structure (upward sloping and with more volatility at long end)

- But, can ask following questions:
  - How does choice of monetary policy affect inflation and nominal term structure?
  - How do EIS and RRA affect inflation and nominal term structure?
  - How does the stochastic vol assumption affect inflation and nominal term structure
How Does Monetary Policy Affect Inflation and Nominal Yield Curve?

- Taylor Rule
  
  \[ i_t = -\ln b_t^1 = \bar{\tau} + \tau_x x_t + \tau_p p_t + s_t \]

- Exogenous \( s_t \), AR(1), orthogonal to shocks driving \( x_t \) and \( v_t \)

- Implied inflation
  
  \[ p_t = \bar{\pi} + \pi_x x_t + \pi_v v_t + \pi_s s_t \]

- Making nominal rate more sensitive to output growth (larger \( \tau_x \)) \( \rightarrow \) inflation more sensitive to stochastic vol (larger \( \pi_v \)) \( \rightarrow \) nominal yield curve shifted down and more upward sloping & higher nominal yield volatility

- Making nominal rate more sensitive to price level (larger \( \tau_p \)) \( \rightarrow \) inflation more sensitive to output growth (larger \( \pi_x \)), less sensitive to stochastic vol (smaller \( \pi_v \)) \( \rightarrow \) nominal yield curve shifted down and more upward sloping \( \rightarrow \) higher nominal yield volatility \( \rightarrow \) nominal yield curve shifted down and less upward sloping & less nominal yield volatility
Intuition

• More intuition for Taylor $\rightarrow$ Implied inflation
• Existing intuition: Implied Inflation $\rightarrow$ nominal SPD $\rightarrow$ nominal yield curve

\[ b_t^n, \$ = b_t^n E_t[P_t/P_{t+n}] \left[ 1 + \text{Cov}_t \left( \frac{m_t/m_{t+n}}{E_t[m_{t+n}/m_t]}, \frac{P_t/P_{t+n}}{E_t[P_t/P_{t+n}]} \right) \right] \]

• Add extra step: Implied Inflation $\rightarrow$ nominal SPD $\rightarrow$ inflation risk premium $\rightarrow$ nominal yield curve
How Does EIS Affect Inflation and Nominal Yield Curve?

- This has **both** real and nominal effects!
- **Real effects**
  - Higher EIS → less demand for long-term bonds, higher real long-term yields → real term structure less downward sloping & Δ of real yields wrt output growth is lower → less volatile real long-term yields [But higher EIS → lower real risk-free rate (Weil)?]
- **Nominal effects**
  - Higher EIS → more upwards sloping nominal term structure % more volatile nominal long-term yields more demand for bonds, lower long-term yields
Questions/Comments

- Stochastic vol seems very important to get upwards sloping nominal term structure.
- Would be instructive to see model without stochastic vol in consumption growth.
- For the same parameters that generate reasonable nominal term structure dynamics, can you get a reasonable equity risk premium?
- Link autocovariance of nominal SDF to term structure of inflation risk premium.
- How different are the statistical properties of implied inflation from data?
- Can you classify the family of monetary policy rules that gives realistic implied inflation?
Extensions

- With existing model, look at
  - Equity Risk Premium
  - Bond Options and their Greeks (see how they respond to macro factors and Taylor Rule)
  - Corporate Bonds: how does monetary policy affect risk-neutral default probabilities?