Portfolio Choice with Model Misspecification
A Foundation for Alpha and Beta Portfolios
Discussion

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Aims (& some Notation) I

- How do you structure a static portfolio when
  - you don’t know the correct factors – what they are and how many there are.
  - you don’t know the correct model for expected risk premia, but you assume they are decomposed into factor-independent and factor-dependent components
  - there is no asymptotic arbitrage
Aims (& some Notation) II

- linear factor model

\[(1) \quad r_t = \mu_N + B_N \begin{pmatrix} z_t \end{pmatrix}_{N \times K} + \epsilon_t\]

- orthogonal $z_t$ and $\epsilon_t$

\[(2) \quad z_t \sim (0, \begin{pmatrix} \Omega \end{pmatrix})\]
\[(3) \quad \epsilon_t \sim (0, \begin{pmatrix} \Sigma \end{pmatrix})\]

- vector of pricing errors

\[(4) \quad \tilde{\alpha}_N = \mu_N - r_f 1_N - B_N \tilde{\lambda}_t\]

- vector of risk premia

\[(5) \quad \tilde{\lambda} = (\tilde{B}_N^T \Sigma^{-1} \tilde{B}_N)^{-1} \tilde{B}_N \Sigma^{-1} (\mu_N - r_f 1_N)\]

- constraint

\[(6) \quad \forall N \exists \text{ finite } \delta : \tilde{\alpha}_N^T \Sigma^{-1} \tilde{\alpha}_N \leq \delta\]
A sequence of portfolios is said to generate an asymptotic arbitrage opportunity if along some subsequence $N'$: $\text{var}(\mathbf{r}_t \mathbf{w}_{N'}^a) \to 0$ as $N' \to \infty$ and $(\mu_{N'} - r_f \mathbf{1}_{N'})^\top \mathbf{w}_{N'}^a \geq \delta > 0 \ \forall \ N'$. 
Why do we care?

Do you know the factors which expected risk premia load on?

If not, you will have to deal with **model misspecification**

- Pricing errors unrelated to factors
  - Managerial skill and analyst recommendation
  - Subjective views

- Pricing errors related to factors
  - Incorrect means/risk premia or covariances for the factors
  - Missing factors (only some factors are observed)
  - Mismeasured factors (Roll critique).

How does dealing with model misspecification in the above model (APT) impact portfolio choice?
Existing theory makes strong and in my view implausible assumptions about covariance matrix for errors. For example, consider a very simple covariance matrix

\[
\Sigma_N = \sigma^2 [\rho J_N + (1 - \rho) I_N]
\]

- first \( N - 1 \) e-values are repeated root \( 1 - \rho \) and dominant e-value is \( 1 + \rho(N - 1) \)
- dominant e-value clearly unbounded as \( N \to \infty \)
- paper overcomes this weakness
Challenges I

- Existing portfolio choice approach stresses factors and estimation of expected factor risk premia
  - Implementation is hard because expected risk premia are hard to estimate
- This paper tells us to focus on estimating pricing errors instead
Decompose mean-variance portfolio into alpha and beta portfolios

\( w^{mv} = \delta^a w^\alpha + (1 - \delta^a) w^\beta \)

- \( \alpha \) portfolio \( w^\alpha \) - combination of long and short positions dependent on pricing errors
- \( \beta \) portfolio \( w^\beta \) - long portfolio dependent on factor risk premia

alpha portfolio more important (for large \( N \))

beta portfolio is a second order issue (for large \( N \)) – if we focus on this, it is a bit like colouring a an outline, without really figuring how sensible the outline is.

- have created a portfolio immune to beta misspecification
- change focus of empirical portfolio choice from beta estimation to pricing error estimation
- obtain higher out of sample Sharpe ratios
Can you show that most covariance matrices have unbounded dominant e-values?

I found it hard to construct examples where the dominant e-values of covariance was bounded.

Can you prove that 'most' (in a sense to made precise) covariance matrices have an unbounded dominant e-value?
Can you relate your work to the FF portfolios?

- In your decomposition alpha portfolios look like zero cost portfolios and the beta portfolio looks close to the market portfolio.
- HML and SMB portfolios are zero cost – made of long and short positions like pricing error portfolios appear to be.
- Do positions in the HML and SMB portfolios correspond to the pricing-error part of your portfolio decomposition?
In robust control, we augment the objective function with a penalty.

The constraint on pricing errors is essentially a penalty – is it related to the Kullback-Leibler divergence?

- I am pretty sure it is, which means that we can think of a particular pricing error as defining a probability measure. Each probability measure defines a model.
- But how does the penalty impact portfolio choice?
Dynamic portfolio choice

- From Merton we know that there is an intertemporal hedging demand component in the optimal portfolio when there is a stochastic investment opportunity set.
- How is this hedging demand portfolio impacted by pricing errors and how they vary over time?
Equilibrium implications of your portfolios

- If investors hold portfolios with a pricing error component and risk-factor component, what does that imply for asset returns once you impose market clearing?
- Does it mean that asset prices are driven mainly by pricing errors, which could generated by differences in beliefs?
- Should we shift the focus of asset pricing towards differences in beliefs and learning? Maybe this is why some hedge funds don’t hire financial economists (Renaissance)
Summary

- Paper with important implications
- Shifts focus of empirical portfolio choice away from beta estimation to pricing error estimation
- Could shift focus of asset pricing towards what drives pricing errors as opposed to betas.
- Need to set up a fund to see how well it works on non-simulated data