Robust Assessment of Hedge Fund Performance through Nonparametric Discounting by Almeida and Garcia

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Do hedge funds really provide risk – adjusted returns much higher than the risk - free rate?

Or is the risk – adjustment inaccurate

Will a better risk adjustment lead to lower risk – adjusted returns?

How can we carry out the risk adjustment more accurately?
Overview II

Theoretical Asset Pricing

Definition 1

\( m \) is a SDF if

\[ \forall \text{ assets } i, \ E[mR_i] = 1. \]  \hfill (1)

Empirical Asset Pricing

- Using some empirical methodology [e.g. Hansen & Jagannathan, (1991)], find that

\[ m = a + \sum_{j=1}^{J} b_j f_j \]  \hfill (2)

- factor, usually a return of a basis asset

- Using empirically determined SDF, price hedge fund return

\[ E[mR_{HF}] = \alpha \]  \hfill (3)

- \( \alpha \) large and positive

- hedge fund offers high risk adjusted return or the SDF has been determined inaccurately
This paper

- New methodology for determining a family of SDF’s from time series data on returns
- SDF can be a non linear function of factors (returns on basis assets)
  - usual approach: include options in set of basis assets, keep \( m \) linear
- relative to linear SDF, non linear SDF’s give lower \( \alpha \)
  - some of the non linear SDF’s capture better higher moments – reduction in \( \alpha \)
  - accounting for extreme events leads to a reduction in \( \alpha \)
- determine SDF via Fenchel dual problem – easier than primal approach
  - dual problem is a portfolio choice problem
Estimating a SDF I

- for all assets, $i$

$$ E[mR_i] = 1 $$

- this is an average over states of the economy

$$ \sum_{k=1}^{K} p(\omega_k) m(\omega_k) R_i(\omega_k) = 1 $$

- if Ergodic Hypothesis is true (replace average over states with time series average – can use time series data), then

$$ \lim_{T \to \infty} \frac{\sum_{t=1}^{T} m_t R_{i,t}}{T} = E[mR_i] $$

average over states

- time series average

- For large $T$

$$ \sum_{t=1}^{T} \frac{m_t R_{i,t}}{T} = 1 $$

$$ \frac{1}{T} \sum_{t=1}^{T} m_t (R_{i,t} - a^{-1}) = 0 $$

where $a = \sum_{t=1}^{T} \frac{m_t}{T}$

- Need an extra condition to determine SDF
Estimating a SDF II


\[
\hat{m}_{HJ} = \frac{1}{2} \arg\min_{\{m_t\}_{t=1}^T} \frac{1}{T} \sum_{t=1}^T [(m_t)^2 - a^2]
\]

\[
\text{s.t.}
\]

\[
\forall i \in \{1, \ldots, I\}, \quad \frac{1}{T} \sum_{t=1}^T m_t(R_{i,t} - a^{-1}) = 0
\]

\[
\sum_{t=1}^T \frac{m_t}{T} = a
\]

\[
\forall t \in \{1, \ldots, T\}, \quad m_t > 0.
\]


- This paper: replace (8) by \( \hat{m}_{MD} = \frac{1}{2} \arg\min_{\{m_t\}_{t=1}^T} \frac{1}{T} \sum_{t=1}^T \phi(m_t) \), where \( \phi(\cdot) \) is a convex function of the form

\[
\phi^\gamma(m) = \frac{m^{1+\gamma} - a^{1+\gamma}}{\gamma(1+\gamma)}
\]
Fenchel dual problem

Instead of finding \( \{m_t\}_{t=1}^T \) find \( \{\lambda_i\}_{i=1}^I \): \( I < T \) (easier)

\[
\hat{\lambda} = \underset{\epsilon \in \mathbb{R}, \lambda a}{\text{arg sup}} \frac{1}{T} \sum_{t=1}^T \psi^\gamma (\epsilon + \lambda T (R_t - \frac{1}{a} 1_I)) \tag{13}
\]

\[
\psi^\gamma (z) = \sup_{w > 0} zw - \phi^\gamma (w) \tag{14}
\]

\[
\hat{m}_{t, MD} = a \left( a^\gamma + \gamma \hat{\lambda}^T (R_t - \frac{1}{a} 1_I) \right)^{1/\gamma} \frac{1}{T} \sum_{t=1}^T \left( a^\gamma + \gamma \hat{\lambda}^T (R_t - \frac{1}{a} 1_I) \right)^{1/\gamma} \tag{15}
\]

- SDF is non linear

- Prior literature: SDF linear but with returns from assets with non linear payoffs

- \( \gamma = 1 \): reduces to linear SDF. Hansen – Jagannathan (1991) is a special case of this paper.

- Dual problem can be interpreted as a portfolio choice problem

- investor with HARA utility: \(-\gamma\) cautiousness parameter

- \( \{\lambda_i\}_{i=1}^I \) are units of wealth invested in each risky asset
What is going on?

- Researcher chooses a set of basis assets: \( \{R_i\}_{i=1}^l \)
- HARA investor with cautiousness, \(-\gamma\) chooses portfolio of the basis assets, i.e. \(\lambda\)
- \(\lambda\) plus the basis assets gives us the nonlinear SDF, \(m\)
- for each \(\gamma\) get a different SDF: a family of SDF’s
Family of SDF’s

Portfolios ($\lambda$)
- More cautious investors ($-\gamma > 0$ [see Eq (14) in paper]) and larger: put more weight on skewness and kurtosis
- $-\gamma > 0$ and larger: more weight on skewness and kurtosis in SDF
- returns: S&P 500 (equity), RU2000 (equity, size – spread), MSCI (emerging market index), 10yrTr, BAA, $r_f$
  - all investors: short BAA, long all others
  - more cautious investors: more weight on $r_f$ rises, less weight on risky assets

SDF volatility $\equiv$ market price of risk
- options added to set of basis assets: higher market price of risk and more sensitivity to extreme events
- More cautious HARA investor: higher market price of risk
- higher market price of risk: will lower performance of any risky strategy correlated with SDF
- extreme events v. important for correct analysis of hedge fund performance if HF’s sell puts!

Pricing errors
- Lowest for $-\gamma = 3, 2, 0$
- Large for $-\gamma = 1$ (HJ)
Hedge fund performance

- Increasing cautiousness of HARA investor: SDF more volatile, lower pricing errors.
- Adding options to basis assets: SDF higher during very bad states of the world and SDF more volatile.
- Increasing cautiousness of HARA investor and adding options to basis assets lowers hedge fund performance.
Suggestions: Fenchel duality

- Fenchel duality appears in many places: physics (with very clear interpretations), finance (with less clear interpretations)
  - Consumption – portfolio choice problems with constraints: more easily solved using Fenchel dual [Cvitanic & Karatzas (1992)]
  - Social planner’s optimization problem where individual agents have recursive preferences: more easily solved using Fenchel dual [Dumas, Uppal & Wang (2000)]
    - Fenchel dual can be interpreted as problem of a robust investor
- This paper: why does the Fenchel dual problem in this paper appear in the form of a portfolio choice problem
- Can we find a unified economic interpretation for Fenchel dual problems in general?
Suggestions: Testing the methodology

- Compare your methodology for estimating SDF with others in a framework where we actually know what the SDF is:
  - consumption-based asset pricing model, complete markets
    - e.g., $N$ dividend trees subject to rare disasters, single agent with power utility or several agents with catching up with Jones preferences (different curvature parameters), complete markets
  - compute unique SDF, $m$, directly
  - use simulated time series data on returns to estimate SDF, $\hat{m}$: measure $||\hat{m} - m||$
  - consumption-based asset pricing model, incomplete markets
    - each agent’s SDF can be computed directly, $m_h$
    - use simulated time series data on returns to estimate SDF, $\hat{m}$: which SDF is it closest to?