Outline

- Aim
- Why do we care?
- Model Summary
- Questions & Suggestions
Main question: How does monetary policy impact how much banks lend to firms and households?

Traditional approach to monetary policy in macro models involves
- ignoring banks
- ignoring risk premia

Some work now incorporates risk premia – Palomino (2010), Bhamra, Fisher & Kuehn (2011); Gomes, Jermann & Schmid (2013).

Not too much on banks. This paper slots banks into a macro model.

Why do we care about all of this?
Why do we care?

- Banking is not the boring and stable industry it once was.
  - Woodford (2010): Financial Intermediation can be subject to frictions.

Stylized Fact from Financial Crisis

- Fed decreased interest rates and increased supply of reserves hoping to stimulate lending by commercial banks.
- Commercial banks allocated more wealth to reserves, but did not allocate much more to loans.
Figure: Fed Assets

Federal Reserve Assets

Other Assets
Liquidity Facilities
MBS+Agency
Treasury Securities

US$ Billion

Jan03 Jan05 Jan07 Jan09 Jan11

Lehman + AIG
Bear Stearns
Figure: Loans

Commercial and Industrial Loans

Actual (with correction for loan commitments)
Actual

US$ Trillion

Sep07, Dec07, Mar08, Jun08, Sep08, Dec08, Mar09, Jun09, Sep09, Dec09, Mar10, Jun10
Model Outline

- **Step 1:** Partial equilibrium portfolio choice approach. Banks receive cash deposits and choose how to allocate balance sheet wealth between reserves and loans. Banks like high expected returns on wealth, but dislike variance.
- **Step 2:** Introduce bank heterogeneity via different stochastic variation in deposits.
- **Step 3:** Compute net demand for reserves (and net supply of loans)
- **Step 4:** Central Bank decides supply of reserves
- **Step 5:** Market clearing
- **Step 6:** Pretend to be a Central Banker: play with the model by changing supply of reserves.
Model Summary

Bank Model (with simplifications)

- Receive cash deposits
  - Cash deposits can be withdrawn wholly or in part at any time – Bank has written a perpetual American call. Date-\(t\) real value of these liabilities is \(D_t\).
- Invest some of deposit cash in form of loans to firms. Loans are illiquid. Date-\(t\) real value of loans is \(B_t\).
- Hold some of deposit cash as reserves. Reserves are liquid. Date-\(t\) nominal value of loans is \(C_t\). Real value is \(p_t C_t\). Reserves need to keep pace with deposits.
  \[
  = \frac{C_t}{p_t}
  \]
- Date-\(t\) real value of equity, \(E_t\)
  \[
  E_t = p_t C_t + B_t - D_t
  \]
- Real return on equity
  \[
  \frac{E_{t+1}}{E_t} = \left(1 - \frac{\text{Div}_t}{E_t}\right) \left[ w_{c,t} R_{t+1}^C + w_{b,t} R_{t+1}^B - (w_{c,t} + w_{b,t} - 1) R_{t+1}^D - R_{t+1}^X (w_{c,t}, w_{d,t}) \right]
  \]
  \[
  = \Omega_{t+1}
  \]
\[ \Omega_{t+1} = R_{t+1}^b + w_{c,t}(R_{t+1}^c - R_{t+1}^b) + w_{d,t}(R_{t+1}^b - R_{t+1}^d) - R_{t+1}^\chi(w_{c,t}, w_{d,t}) \]

\[ w_{d,t} \leq \kappa \]

- \( R_{t+1}^\chi(w_{c,t}, w_{d,t}) \): **liquidity cost** reflecting difference between interest charges on borrowing reserves (when in deficit after receiving more deposits) and depositing excess reserves

- Bank maximizes power utility over intermediate consumption

\[
u(Div_t) = \frac{Div_t^{1-\gamma}}{1-\gamma}
\]

\[
V_t = \max_{\{Div_t, w_{c,t}, w_{b,t}\}} \underbrace{u(Div_t)} + \beta E_t[V_{t+1}]
\]

(3)

- homotheticity \( \Rightarrow V_t = \nu_t E_t^{1-\gamma} \). Define \( div_t = \frac{Div_t}{E_t} \)

\[

\nu_t = \max_{\{div_t\}} u(div_t) + \beta (1 - div_t)^{1-\gamma} \max_{\{w_{c,t}, w_{b,t}\}} E_t[\nu_{t+1} \Omega_{t+1}^{1-\gamma}] 
\]

(4)

- Standard consumption-portfolio choice problem + **liquidity cost** (no-riskfree asset)
  - mean-variance portfolio + hedging demand
Model-Central Bank

Most interesting and strangest part of the model. Central Bank has no objective function. Just varies parameters/variables.

- Via interest rates: indirectly controls liquidity costs which help determine risk premia
- Via money supply, controls net supply of reserves (market clearing)

\[
\text{Demand for reserves} = \text{Supply of reserves} \\
\text{bank portfolio choice problem} = \text{Central Bank (5)}
\]
**Banks & Expected Risk Premia**

- Loans are illiquid

\[
R_{t+1}^b - R_{t+1}^c = E_t[R_{c,t+1}^\chi] - \text{Cov}_t \left[ \frac{M_{t+1}}{E_t[M_{t+1}]}, R_{c,t+1}^\chi \right]
\]  

(6)

- Reserves are liquid

\[
R_{t+1}^d - R_{t+1}^c = E_t[R_{d,t+1}^\chi] - \text{Cov}_t \left[ \frac{M_{t+1}}{E_t[M_{t+1}]}, R_{d,t+1}^\chi \right]
\]  

(7)
Main Question Again

- **Old statement**: How does monetary policy impact how much banks lend to firms and households?

- **New statement**: How does monetary policy impact expected risk premia, volatilities and correlations and hence the proportion of bank balance sheets allocated to loans

- This is about Central Banks and Asset Pricing.
  - Central Banks are suppliers of assets.
  - Commercial Banks are purchasers of assets.
- Market incompleteness
- Central Bank Tools and Expected Risk Premia
- Hedging Demand
- SDF
- Banks and Portfolio Choice
- Real economy
- Central Bank’s Objective Function
The only way the Central Bank can alter the risk-return relationship for assets is if markets are incomplete.

- Explain how markets are incomplete in this model.
- Seems to stem from difference in interest rate Fed charges when lending reserves (high) and accepting reserve deposits (low)
- Why is the Fed creating this friction in the first place?
The Central Bank can do many things to alter risk-return relationship for assets. Why not analyze their effects one-by-one on expected risk premia etc?

- Altering reserves requirement
- Altering interest rates on reserves

Then see how this maps into portfolio weights.

Need to check expected risk premia are realistic in magnitude (hard with power utility, EZW?)
Questions - Hedging Demand

\[ \gamma = 0.5 \neq 1. \] Part of portfolio demand stems from hedging demand.

- How large is hedging demand relative to myopic demand?
- Is the reserve asset being used to hedge against fluctuations in some state variable? Is this why banks hold it?
SDF key for determining expected risk premia. What happens to its mean and variance as Central Bank

- Alters reserves requirement
- Alters interest rates on reserves

Then see how this maps into expected risk premia
Who is the marginal investor pinning down the SDF?

In the model appears to be only banks who consume. Is modelling banks as consumers ok?

In the world, households consume.

Is the marginal investor really a bank?
Banks do more than just take deposits and lend.

They are active in asset markets - CDO’s, derivatives, etc

Changing the risk-return tradeoff for a bank is not so easy with more assets: markets less incomplete.

Change model to reflect this: more financial assets?
Suppose central banks could induce a large enough increase in expected risk premia on loans so that commercial banks lend more.

We don’t care about more lending per se.

We only care if more lending leads to an increase in trend output.

Need a production sector.
Suggestions- Central Bank’s Objective Function

Is the Central Bank just a parameter twiddler or does the twiddling have an aim?

- With consuming households could try and maximize their welfare?
  - Do this in a production economy?
  - Even better? Endogenous growth model.

- Are Central Banks in the business of trying to complete markets?
  - Incomplete $\rightarrow$ Complete: raises welfare
  - Incomplete $\rightarrow$ Less incomplete: anything could happen to welfare
Dynamic Budget Constraint

Denote the date-\(t\) nominal value of reserve assets held by the Bank via \(C_t\). Hence, the date-\(t\) real value of reserve assets is \(C_t^r = p_t C_t\), where \(p_t\) is the reciprocal of the price index, i.e. \(p_t = \frac{1}{P_t}\).

Denote the date-\(t\) real value of loans via \(B_t\).

Denote the date-\(t\) real value of deposit liabilities via \(D_t\).

Date-\(t\) Bank equity is given by

\[
E_t = C_t^r + B_t - D_t, \quad (8)
\]

Date-\(t + dt\) Bank equity is given by

\[
E_{t+dt} = C_{t+dt}^r + B_{t+dt} - D_{t+dt} - C_t^o dt, \quad (9)
\]

where \(C_t^o\) is the date-\(t\) rate at which bank owners consume from the stock of bank wealth. The consumption rate is a flow variable while wealth is a stock variable.

The infinitesimal increment in Bank equity over the interval \([t, t + dt)\) is given by

\[
dE_t = E_{t+dt} - E_t = (C_{t+dt}^r - C_t^r) + (B_{t+dt} - B_t) - (D_{t+dt} - D_t) - C_t^o dt \quad (10)
\]

\[
= dC_t^r + dB_t - dD_t - C_t^o dt \quad (11)
\]
The date-$t$ proportion of bank equity held in reserve assets is $w_{c,t} = \frac{C^t}{E_t}$. The date-$t$ proportion of bank equity held in loans is $w_{b,t} = \frac{B^t}{E_t}$, and the date-$t$ proportion of bank equity held in deposit liabilities is $w_{d,t} = \frac{D^t}{E_t}$. Market clearing implies that $w_{c,t} + w_{b,t} - w_{d,t} = 1$. We have thus rewrite the Bank’s dynamic budget constraint as

$$\frac{dE_t}{E_t} = w_{c,t} \frac{dC^t}{C^t} + w_{b,t} \frac{dB^t}{B_t} - w_{d,t} \frac{dD^t}{D_t} - \frac{C^o_t}{E_t} dt. \quad (12)$$

So far we have only included capital gains, but we can easily include any dividend or coupon payments by writing

$$\frac{dE_t}{E_t} = w_{c,t} dR^C_t + w_{b,t} dR^B_t - w_{d,t} dR^D_t - \frac{C^o_t}{E_t} dt, \quad (13)$$

where $dR^C_t$ is the instantaneous cum-dividend or cum coupon return on reserve assets, and so on. Usually the risk-free asset acts as a reference asset, with respect to which measure risk premia. In this case, the safest asset, the reserve asset plays this role. Accordingly, we write

$$\frac{dE_t}{E_t} = (1 - w_{b,t} + w_{d,t})dR^C_t + w_{b,t} dR^B_t - w_{d,t} dR^D_t - \frac{C^o_t}{E_t} dt \quad (14)$$

$$= dR^C_{t,t} + w_{b,t}(dR^B_t - dR^C_t) - w_{d,t}(dR^D_t - dR^C_t). \quad (15)$$
Modelling returns

We assume the instantaneous return on the reserve asset has a constant expected rate of return together with an unexpected component which is driven by the increment of a standard Brownian motion, i.e.

$$dR_{C,t} = \mu_c dt + \sigma_c dZ_{c,t}.$$  \hfill (16)

Similarly, we model the excess returns on loans and liabilities via

$$dR_{t}^B - dR_{C,t} = \mu_b dt + \sigma_b dZ_{b,t}$$ \hfill (17)

$$dR_{t}^D - dR_{C,t} = \mu_d dt + \sigma_d dZ_{d,t}.$$ \hfill (18)

Note that $dZ_{c,t}dZ_{b,t} = \rho_{cb} dt$, $dZ_{b,t}dZ_{d,t} = \rho_{bd} dt$, $dZ_{c,t}dZ_{d,t} = \rho_{cd} dt$.

It is important to note that the investment opportunity set is constant (all expected returns, expected excess returns, volatilities and correlations are constant).

Therefore, the expected instantaneous return on bank equity is

$$\mathbb{E}_t \left[ \frac{dE_t}{E_t} \right] = (\mu_c + w_{b,t}\mu_b - w_{d,t}\mu_d) dt - \frac{C^o_t}{E_t} dt$$ \hfill (19)

and the variance of the instantaneous return is

$$\text{Var}_t \left[ \frac{dE_t}{E_t} \right] = (\sigma_c^2 + w_{b,t}^2\sigma_b^2 + w_{d,t}^2\sigma_d^2 + 2w_{b,t}\rho_{cb}\sigma_c\sigma_b - 2w_{d,t}\rho_{cd}\sigma_c\sigma_d - 2w_{b,t}w_{d,t}\rho_{bd}\sigma_b\sigma_d) dt$$ \hfill (20)
Bellman eqn and HJB de

The bank chooses its consumption flow and portfolio policies to maximize expected utility from intermediate consumption, i.e.

$$V_t = \sup_{(C_u^0, w_{b,u}, w_{d,u}) \in [t, \infty)} E_t \int_t^\infty e^{-\beta(u-t)} U(C_u^0) du.$$  \hspace{1cm} (21)

We have the Bellman equation

$$V_t = \sup_{(C^o_t, w_{b,t}, w_{d,t})} U(C^o_t) dt + e^{-\beta dt} E_t [V_{t+dt}].$$  \hspace{1cm} (22)

Applying Ito’s Lemma gives the Hamilton-Jacobi-Bellman differential equation

$$0 = \sup_{(C^o_t, w_{b,t}, w_{d,t})} U(C^o_t) - \beta V_t + E_t \frac{\partial V_t}{\partial E_t} \frac{1}{E_t} \mathbb{E}_t \left[ \frac{dE_t}{E_t} \right] + \frac{1}{2} E_t^2 \frac{\partial^2 V_t}{\partial E_t^2} \frac{1}{E_t} \text{Var}_t \left[ \frac{dE_t}{E_t} \right]$$  \hspace{1cm} (23)

Using our expressions for expected bank equity returns and the variance of bank equity returns, we obtain

$$0 = \sup_{(C^o_t, w_{b,t}, w_{d,t})} U(C^o_t) - \frac{\partial V_t}{\partial E_t} C^o_t - \beta V_t + E_t \frac{\partial V_t}{\partial E_t} (\mu_c + w_{b,t} \mu_b - w_{d,t} \mu_d)$$  \hspace{1cm} (24)

$$+ \frac{1}{2} E_t^2 \frac{\partial^2 V_t}{\partial E_t^2} \left( \sigma^2_c + w_{b,t}^2 \sigma_b^2 + w_{d,t}^2 \sigma_d^2 + 2w_{b,t} \rho_{cb} \sigma_c \sigma_b - 2w_{d,t} \rho_{cd} \sigma_c \sigma_d - 2w_{b,t} w_{d,t} \rho_{bd} \sigma_b \sigma_d \right).$$  \hspace{1cm} (25)
Separation of Consumption and Portfolio Choice

Rearranging the HJB de

\[ 0 = -\beta V_t + \sup_{C_t^o} U(C_t^o) - \frac{\partial V_t}{\partial E_t} C_t^o \]  

\[ + \sup_{(w_b,t,w_d,t)} E_t \frac{\partial V_t}{\partial E_t} (\mu_c + w_{b,t}\mu_b - w_{d,t}\mu_d) \]  

\[ + \frac{1}{2} E_t^2 \frac{\partial^2 V_t}{\partial E_t^2} (\sigma_c^2 + w_{b,t}^2\sigma_b^2 + w_{d,t}^2\sigma_d^2 + 2w_{b,t}\rho_{cb}\sigma_c\sigma_b - 2w_{d,t}\rho_{cd}\sigma_c\sigma_d - 2w_{b,t}w_{d,t}\rho_{bd}\sigma_b\sigma_d) \]  

\[ (26) \]

\[ (27) \]

\[ (28) \]

So we have the consumption-savings decision

\[ \sup_{C_t^o} U(C_t^o) - \frac{\partial V_t}{\partial E_t} C_t^o \]  

\[ (29) \]

where we trade off the marginal benefits of more consumption today against the marginal cost of saving less as encapsulated in the FOC

\[ U'(C_t^o) = \frac{\partial V_t}{\partial E_t}. \]  

\[ (30) \]
The optimal portfolio decision is given by

\[
\sup_{(w_{b,t}, w_{d,t})} E_t \frac{\partial V_t}{\partial E_t} (\mu_c + w_{b,t} \mu_b - w_{d,t} \mu_d)
\]  

(31)

\[
+ \frac{1}{2} E_t^2 \frac{\partial^2 V_t}{\partial E_t^2} \left( \sigma^2_c + w_{b,t} \sigma_b^2 + w_{d,t} \sigma_d^2 + 2w_{b,t} \rho_{cb} \sigma_c \sigma_b - 2w_{d,t} \rho_{cd} \sigma_c \sigma_d - 2w_{b,t} w_{d,t} \rho_{bd} \sigma_b \sigma_d \right)
\]  

(32)

Note that \( \frac{\partial V_t}{\partial E_t} > 0 \) and by concavity of the value function \( \frac{\partial^2 V_t}{\partial E_t^2} < 0 \), and so the portfolio decision is equivalent to

\[
\sup_{(w_{b,t}, w_{d,t})} (\mu_c + w_{b,t} \mu_b - w_{d,t} \mu_d)
\]  

(33)

\[
- \frac{1}{2} \left( \frac{-E_t^2 \frac{\partial^2 V_t}{\partial E_t^2}}{E_t \frac{\partial V_t}{\partial E_t}} \right) \left( \sigma^2_c + w_{b,t} \sigma_b^2 + w_{d,t} \sigma_d^2 + 2w_{b,t} \rho_{cb} \sigma_c \sigma_b - 2w_{d,t} \rho_{cd} \sigma_c \sigma_d - 2w_{b,t} w_{d,t} \rho_{bd} \sigma_b \sigma_d \right)
\]  

(34)

The bank wants a high expected return on equity but dislikes variance. The penalty attached to variance is given by \( \frac{-E_t^2 \frac{\partial^2 V_t}{\partial E_t^2}}{E_t \frac{\partial V_t}{\partial E_t}} > 0 \).
From homotheticity

\[ V_t = U(AE_t) \]  

for some \( A \). \( A \) will be a constant. Hence, the portfolio choice problem is

\[
(\mu_c + w_{b,t}\mu_b - w_{d,t}\mu_d) - \frac{1}{2}\gamma(\sigma_c^2 + w_{b,t}\sigma_b^2 + w_{d,t}\sigma_d^2 + 2w_{b,t}\rho_{cb}\sigma_c\sigma_b - 2w_{d,t}\rho_{cd}\sigma_c\sigma_d - 2w_{b,t}w_{d,t}\rho_{bd}\sigma_b\sigma_d).
\]

Solving the above problem gives

\[
w_b = \frac{1}{1 - \rho_{bd}^2} \left( \frac{1}{\gamma} \frac{\mu_b - \beta_{bd}\mu_d}{\sigma_b^2} - \beta_{cb} + \beta_{cd}\beta_{db} \right)
\]

\[
w_d = \frac{1}{1 - \rho_{bd}^2} \left( \frac{1}{\gamma} \frac{\beta_{db}\mu_b - \mu_d}{\sigma_b^2} + \beta_{cd} - \beta_{bd}\beta_{cb} \right),
\]

where the cross-asset beta, \( \beta_{ij} \), is defined by

\[ \beta_{ij} = \frac{\rho_{ij}\sigma_i}{\sigma_j}. \]

Potential empirical exercise: measure expected excess returns, volatilities, correlations, cross-asset betas before and after monetary policy changes (event study). See how theoretical portfolio weights change.
Intertemporal Hedging Demand

With a stochastic investment opportunity set, e.g. stochastic expected returns, volatilities or correlations, portfolio demand will contain extra hedging demand terms when $\gamma \neq 1$. This reflects the fact that non-myopic investors care about future expected returns etc. and seek to hedge against possible changes in future expected returns etc.