BKK the EZ Way (Backus-Kehoe-Kydland the Epstein-Zin Way)  
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Outline

- Aim
- Why do we care?
- Model Summary & Results
- Comments
Paper’s aim:

- Design a **international production-based model with endogenous labor**
  - matches a large number of moments from international asset pricing and macro
  - capital does not always flow to most productive country

Key features

- Epstein-Zin preferences
- More home bias in consumption than investment (macro variables)
- Heterogenous productivity of vintage capital (asset prices)
Why do we care?

Improving the world economy

Unlike the stars, the sun etc., the economy is not wholly a feature of nature. Much of the economy would not exist without humans. Human actions impact the economy. The economy is supposed to improve human welfare.

Would like to use general equilibrium models of the world economy to help design welfare improving policies.

- What form should the US and EU trade agreement take? Who gains and by how much. Do some countries gain more than others? Does anyone lose out?
- Should there be a Eurozone and who should be in it? How large are the welfare gains?
- How large are the welfare costs of international business cycles?
- How much should we invest in education? What type of education? Which countries?

Starting point for this: an international ge model, which is reasonably close to both the international macro and asset pricing data.

This paper provides an example of a 2 country production economy, which is close to both the international macro and asset pricing data.
Model Summary

- 2 countries: home and foreign
  - home: good \( X \)
  - foreign: good \( Y \)

- Representative consumer-worker in each country
  - Demand for consumption
  - Supply of labor

- Production technologies: output in a country depends on labor supply from that country, capital stock and exogenous production technology

- Capital accumulation depends on depreciation and investment from home and foreign sources

- Financial markets dynamically complete: competitive eqm obtained as a Pareto efficient allocation from social planner’s problem
Social Planner’s Problem

\[
\begin{align*}
sup \, \mu_0 W_0 + (1 - \mu_0) W_0^* \\
\text{s.t.}
\end{align*}
\]

\[
\begin{align*}
&\text{home cons of gd } X_t + \text{foreign cons of gd } X^*_t + \text{home inv in gd } I_{x,t} + \text{home inv in gd } I_{y,t} \\
&\text{output good } X \\
&\leq F(A_t, K_t, N_t) \quad (2)
\end{align*}
\]

\[
\begin{align*}
&\text{home cons of good } Y_t + \text{foreign cons of good } Y^*_t + \text{foreign inv in gd } I_{x,t} + \text{foreign inv in good } I_{y,t} \\
&\text{output good } Y \\
&\leq F(A^*_t, K^*_t, N^*_t) \quad (3)
\end{align*}
\]

and

\[
\begin{align*}
K_t &\leq (1 - \delta)K_{t-1} + \bar{\omega}_t G(I_{x,t-1}, I^*_{x,t-1}) \quad (4) \\
K^*_t &\leq (1 - \delta)K^*_{t-1} + \bar{\omega}^*_t G(I_{y,t-1}, I^*_{y,t-1}) \quad (5)
\end{align*}
\]

- **controls:** \( X_t, X^*_t, I_{x,t}, I_{y,t}, N_t \) & \( Y_t, Y^*_t, I^*_{x,t}, I^*_{y,t}, N^*_t \)
- **constraints hold at each date** \( t \)
- **Pareto weights are time-varying** \( \mu_1 \neq \mu_0 \): EZ preferences [Dumas, Uppal & Wang]
Epstein-Zin preferences

- Created to
  - disentangle preferences over states from preferences over date
  - distinguish between consumption smoothing over states (risk sharing) and consumption smoothing over time (intertemporal consumption smoothing)
- Used in production-based asset pricing in combination with long-run risks (Bansal & Yaron, Lochstoer & Kaltenbrunner) to
  - keep risk-free rate low
  - need additional features to increase equity risk premium

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Does not help much

- Model: $\text{Corr}(dc_t, dc^*_t) = 0.41 > 0.23 = \text{Corr}(dx^\text{tot}_t, dy^\text{tot}_t)$

- Perfect risk sharing: SDF for each rep agent must equalize for each date and state

$$M_{t+1} = \beta \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t[U_{t+1}^{1-\gamma}]} \right)^{-\left(\gamma - \frac{1}{\psi}\right)}$$ (6)

- Consumption growth across countries more highly correlated than output growth

- Data: $\text{Corr}(dc_t, dc^*_t) = 0.33 > 0.52 = \text{Corr}(dx^\text{tot}_t, dy^\text{tot}_t)$

- In the real world something else is happening: BKK anomaly
EZ preferences and Pareto weights

- Pareto weights: are they deterministic?
- Stationarity? Earlier work
More home bias in consumption than investment

Home agent consumption good aggregator has more weight on good X
Investment aggregator $G$: not so much weight on local investment

$$K_t = (1 - \delta)K_{t-1} + \bar{\omega}_t G(l_{x,t-1}, l_{x,t-1}^*)$$

Impacts macro quantities

- $\text{Corr}(dc_t, dc_t^*) < \text{Corr}(dx_t^{\text{tot}}, dy_t^{\text{tot}})$
- EZ preferences: agents share long-run risks embedded in continuation utilities

$$M_{t+1} = \beta \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-\frac{1}{\psi}} \left( \frac{U_{t+1}}{E_t[U_{t+1}^{1-\gamma}]} \right)^{-\left(\gamma - \frac{1}{\psi}\right)}$$

- agents can equate their SDF’s by keeping their continuation utilities highly correlated
- easier to do when investment home bias is weaker $\rightarrow$ force agents to share risks via investment channel
- SDF’s can line up across dates and states because of continuation utilities, and so consumption does not have to line up as much across dates and states
- more volatile investment growth
- higher stock return vol
- risk premium still small
Heterogenous productivity of vintage capital

Has large impact on asset prices

- continuum of overlapping vintages of capital
- capital stock is a productivity-based weighted average of new and old investments
- older investments more exposed to productivity risk

\[ K_t = (1 - \delta)K_{t-1} + \bar{\omega}_t G(I_{x,t-1}, I_{x,t-1}^*) \] (8)

\[ \bar{\omega}_t = e^{-(1-\phi_0)\frac{1-\alpha}{\alpha}(\Delta a_t - \mu)} \] (9)

evolution of capital stock is stochastic

- higher risk premium
Comments/Suggestions

- Usefulness of EZ preferences well known now
- Focus on explaining economics behind how macro variables are impacted by assumption that there is more home bias in consumption than investment
  - Forcing volatility and correlation out of consumption and into utilities and perhaps investment
  - How large is $\text{Var}_t[U_{t+1}]$
  - How large is $\text{Corr}_t(U_{t+1}, U^*_t)$
  - Might help us understand why cross-country inv growth correlation has wrong sign (-ve instead of +ve)
- Focus on explaining economics behind how asset prices impacted by heterogenous productivity of vintage capital.
  - How does the mechanism differ from investment shocks (Kogan & Papanikolaou)?
Paper looks at many moments

- 27 moments in Table 4
- Remind us why these moments are important
Acknowledge shortcomings of framework

- Paper assumes home output is produced solely by home labor
  - Labor used to make a computer or a shoe is based in more than one country. Foreign outsourcing has increased since the 1970s [Feenstra, JEP, 1998, Integration of Trade and Disintegration of Production in the Global Economy, 1948 google cites]

- Labor cannot migrate across national borders
  - If we are interested in designing policies which are politically feasible, need to include migration
Conclusion

- Very interesting paper
- Less time on EZ and more time on international investment (macro) and capital vintages (asset pricing)
- Pick your favourite moments