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IMITATION IN FINANCIAL MARKETS

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It is believed that trading agents often imitate the behaviour of those around them. In its excessive form this imitation can help lead to large increases or decreases in asset-prices over a small time, often described as bubbles and crashes. In this paper we examine a model in which rational agents repeatedly trade one asset whose price is influenced by supply and demand together with a stochastic noise term. Each agent is able to observe and remember the actions of her nearest neighbours. Furthermore the agents receive private information about the asset-price. We find that profit-maximization implies that agents should to some extent imitate the behaviour of the people around them allowing the use of the Ising Spin Model to investigate agent-agent interactions.

Keywords: Interactions, beliefs.

1. Introduction

In this paper we consider a network of day-traders who all trade one asset. Each period traders signal their actions to their nearest neighbours. Based on the information supplied to them in the previous period by their nearest neighbours, agents can choose their trading strategies for the next period. Each agent also receives private information. This paper shows that when each trader maximises expected profits the optimal strategy is an aggregate of the information she has received from her nearest neighbours and her own private information. The paper then looks at the effect of this behaviour on the asset-price.

2. The Model

Assume the existence of I traders in a network. We represent the traders as points on a graph. Let $N(i)$ denote the set of agents directly connected to agent i by the graph. We call these the nearest neighbours of i. Time is divided up into periods $0, 1, 2, \ldots, t-1, t, \ldots$. The agents are allowed to trade in one asset which has price $p(t)$ in period t.

Each agent can either buy or sell one unit of the asset. This is modelled by saying that each agent can be in one of two states, $s_i \in \{-1, +1\}$, where -1 denotes selling and $+1$ denotes buying. We also have the condition $P\{s_i = +1\} = P\{s_i = -1\}$ for prior probabilities.

For simplicity assume that the agents are day-traders: they trade at time $t-1$ on a price $p(t-1)$. Cash and asset positions are not transferable from one period to the next. The asset-price is determined by the following equation:

$$\frac{p(t) - p(t-1)}{p(t-1)} = \Delta t \frac{\sum_{i=1}^{I} s_i(t-1)}{I} + \sqrt{\Delta t}\sigma\epsilon(t)\,.$$

The proportional change in the price from time $t-1$ to time t is given by the sum of a drift and diffusion term. The drift term is the average state of the network of traders $\sum_{i=1}^{I} s_i(t-1)/I$ multiplied by the size of a time period Δt. Thus when there is an excess of buyers over sellers, the drift term is positive, pushing the price up.

The diffusion term consists of the volatility of the asset σ multiplied by $\sqrt{\Delta t}\epsilon(t)$, where $\{\epsilon(t)\}_t$ are independent identically distributed random variables with zero mean and finite variance, but fatter right-tails than left-tails. We also have that

$$\forall t \epsilon(t) = \frac{\sum_{i=1}^{I} \epsilon_i(t)}{I}\,,$$

where $\{\epsilon_i(t)\}_i$ are independently distributed random variables with zero mean and finite variance. The random variable $\epsilon_i(t)$ is observed in period t by the ith agent and no one else. The observed value of $\epsilon_i(t)$ is represented by e_{it}.

In period $t-1$ each agent knows the price $p(t-1)$ of the asset and trades just after that price is announced. Agents do not know what the price will be in period t. However in period t, when an agent must make a trading decision, that agent is aware of the previous actions of her nearest neighbours.

The set-up of the model owes much to the literature on informational cascades, which are well documented in [1].

3. Profit Maximisation

We now consider the expected profits of an agent in a particular period given her information from the last period. We show that to maximise her expected profits, the agent chooses a strategy, a which is an aggregate of the signals she has received from her nearest neighbours and her own private signal.

At the end of period t, agent j has a profit of

$$(p(t) - p(t-1))a_j(t)\,.$$

Thus the expected profit at time t given the information agent j has at the end of period $t-1$ is

$$\left(\Delta t \frac{\sum_{i \in N(j)} s_i(t-1)}{I} + \sqrt{\Delta t}\sigma \frac{e_{j,t}}{I}\right) p(t-1) E_{t-1} a_j(t)\,.$$

Hence to maximise her expected profits at the end of period t given her information at the end of period $t-1$, agent j chooses $a_j(t)$ according to the following rule:

$$\hat{a}_j(t) = \text{sign}\left[\sum_{i\in N(j)} s_i(t-1) + \sigma\frac{e_{j,t}}{\sqrt{\Delta t}}\right],$$

where the hat denotes that this is the choice of a that optimises the expected profit.

Thus we have shown that each agent in the market maximises her expected profits by carrying out the aggregate of the signals of her nearest neighbours and her own weighted private signal, $e_{j,t}$. The above expression is also exactly that which appears in the Ising spin model. Thus we have an argument based on economic considerations which shows that use of the Ising spin model for studying agent interactions as documented in [4] is indeed justified.

Attempts to model trading decisions and price formation are not new and very instructive approaches can be found in [3] and [5].

4. The Asset Price

We need to investigate what effect profit maximisation has on prices. We do this by first deriving the distribution of the optimal strategy and seeing how it varies with the asset price volatility σ. Then we use this information to look at the distribution of the drift term, which is the main factor affecting price changes.

The first step is to introduce a graph to describe the network of traders. The simplest one is a $1-d$ circle, with an agent at each point.

Label the agents as follows, s_i, where $i \in \{1,\ldots,N+1\}$. We imagine the $N+1$ agents lying equidistant on a line. To avoid problems at the ends we fold the line into a circle. This way each agent has two nearest neighbours and we are left with N agents.

Consider a typical agent s_i and her nearest neighbours:

$$\begin{array}{c} s_{i-1} \\ s_i \\ s_{i+1} \end{array}$$

First consider the situation where each agent maximises her expected profits.

We need to look at the distribution of $\hat{a}_i(t)$, which will differ from that of its prior. It is seen that

$$\begin{aligned} P(\hat{a}_i(t-1)\le 0) = P(s_{i-1}(t-1)\le 0, s_{i+1}(t-1)\le 0)P\left(\epsilon_{i,t}\le\frac{2\sqrt{\Delta t}}{\sigma}\right) \\ + P(s_{i+1}(t-1)\le 0, s_{i-1}(t-1) \\ + P(s_{i-1}(t-1)\ge 0, s_{i+1}(t-1)\ge 0)P\left(\epsilon_{i,t}\le\frac{-2\sqrt{\Delta t}}{\sigma}\right) \end{aligned}$$

$\epsilon_i(t)$ is a continuous random variable, so $P(\hat{a}_i(t) = 0) = 0$.

Thus

$$P(\hat{a}_i(t) = -1) = \frac{1}{4}\left(P\left(\epsilon_{i,t} \leq \frac{2\sqrt{\Delta t}}{\sigma}\right) + P\left(\epsilon_{i,t} \leq \frac{-2\sqrt{\Delta t}}{\sigma}\right)\right) + \frac{1}{2}P(\epsilon_{i,t} \leq 0)\,.$$

Hence

$$P(\hat{a}_i(t) = +1) = \frac{1}{4}\left(P\left(\epsilon_{i,t} > \frac{2\sqrt{\Delta t}}{\sigma}\right) + P\left(\epsilon_{i,t} > \frac{-2\sqrt{\Delta t}}{\sigma}\right)\right) + \frac{1}{2}P(\epsilon_{i,t} > 0)\,.$$

Note that the only variable which these probabilities depend on is σ and that ϵ_i is asymmmetric.

As σ increases, $P(\epsilon_{i,t} \leq \frac{2\sqrt{\Delta t}}{\sigma})$ decreases and $P(\epsilon_{i,t} \leq \frac{-2\sqrt{\Delta t}}{\sigma})$ increases. If $\epsilon_{i,t}$ has a fatter right-tail than left-tail, then $P(\hat{a}_i(t) = -1)$ will increase as σ increases. Thus as σ increases, the probability that an agent will sell the asset increases.

Define

$$p_i = P(\hat{a}_i(t) = -1)\,.$$

Now consider the distribution of $\sum_i \hat{a}_i$. For simplicity assume N is odd.

We can then easily find $P(\sum_i \hat{a}_i \leq 0)$:

$$P\left(\sum_i \hat{a}_i \leq 0\right) = \sum_{k>N/2}\sum_{j_1\cdots j_k}\prod_{i=1}^{k} p_{j_i} \prod_{j\in\{1,\ldots,N\}/\{j_1,\ldots,j_k\}} (1-p_j)\,.$$

This gives the probability of the drift term of the asset being weakly negative.

For simplicity assume that p_i is independent of i so that $p = p_i$. Thus

$$P\left(\sum_i \hat{a}_i \leq 0\right) = \sum_{k>N/2}\binom{N}{k} p^k (1-p)^{N-k}\,.$$

For $N = 1001$, we investigate the dependence of $P(\sum_i \hat{a}_i \leq 0)$ on p numerically.

Referring to the graph below, we note that over a small band of p-values $[P, Q]$, including $1/2$, $P(\sum_i \hat{a}_i \leq 0)$ increases rapidly from close to zero to very close to 1. Call the range $[P, Q]$, the critical range.

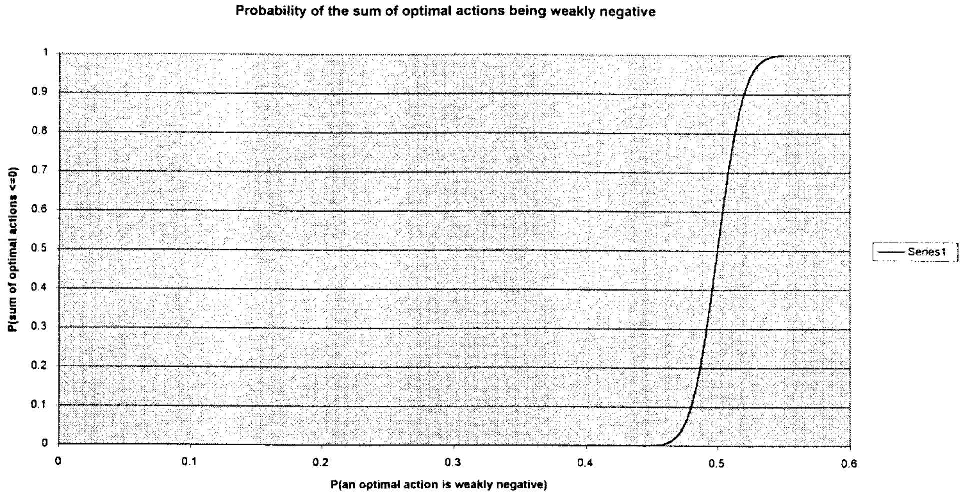
Thus if in a group of homogeneous traders, the probability of a typical trader selling an asset comes close to $1/2$, the probability of the asset price drifting down increases markedly. This probability is close to one, when the probability of a typical trader selling the asset goes above $1/2$. So the behaviour of the asset price is very sensitive to the probability distribution over a small range around $1/2$ called the critical range and is not at all sensitive elsewhere.

Suppose now we have two types of agent, i and j one with

$$P(\hat{a}_i(t) = -1) = p_1$$

the other with

$$P(\hat{a}_j(t) = -1) = p_2\,.$$



There are $[N/2]+1$ agents of the first type and $[N/2]$ of the second type. Without loss of generality, assume that $p_1 > p_2$. Thus the type-1 agents are the more pessimistic. Then

$$P\left(\sum_i \hat{a}_i \le 0\right) = \sum_{k>N/2}\sum_{0\le r\le k}\binom{[N/2]}{r}\binom{[N/2]+1}{k-r} \times p_1^r(1-p_1)^{[N/2]-r}p_2^{k-r}(1-p_2)^{[N/2]+1-k+r}\,.$$

As the asset volatility increases p_1 and p_2 both increase, but p_1 enters the critical range first, increasing the probability of the asset price drifting downwards. Thus the type-2 agents are also likely to sell the asset. The pessimism of the type-1 agents has been transferred to the type-2 agents. Similar conclusions concerning the convergence of beliefs have been suggested by De Marzo, Vayonos and Zwiebel [2].

5. Summary

We have shown that imitative behaviour follows from profit maximisation and furthermore that the use of the Ising spin model to model agent-agent interactions can be justified on economic grounds. An extensive empirical test of the application of the Ising spin model to financial markets is carried out in [4].

Among a group of homogeneous agents, the probability distribution of the traders's optimal actions only influences the asset price evolution in a certain critical range. Outside this range, the asset price distribution is very stable. When there are two groups of traders and one is more optimistic than the other, the beliefs of the group which is affecting the asset price movements more will be imitated by the other group. In effect, we see a convergence of beliefs.

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