Discussion: Generalized Disappointment Aversion and Asset Prices
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1 Motivation

▶ Equity premium puzzle – Mehra and Prescott (1985)

1.1 State-dependent and countercyclical risk aversion

▶ For a representative agent economy to match historical data require higher risk aversion in recessions (i.e. state-dependent and countercyclical risk aversion) – Melino and Yang (2002), Gordon and St-Amour (2000).

▶ Examples:

- Habit formation – Campbell and Cochrane (1999).
- Time varying loss aversion – Barberis, Huang and Santos (2001).
2 Major Contributions

▶ Defining a new set of preferences, exhibiting state-dependent, counter-cyclical risk aversion.

▶ Showing how this can resolve the equity premium puzzle.
3 Generalized Disappointment Aversion

Specific example of recursive preferences – Epstein and Zin (1989).

\[
\text{Recursive class} \begin{cases} \text{Dekel – Chew class} \end{cases} \begin{cases} \text{GDA} \end{cases} \begin{cases} \text{DA} \end{cases} \begin{cases} \text{Kreps – Porteus utility} \\ \text{Standard additive utility} \end{cases}
\]
• Aggregator

\[ W(c, z) = \left[ \left( 1 - \frac{1}{1 + \rho} \right) c^\gamma + \frac{1}{1 + \rho} z^\gamma \right]^{1/\gamma}. \]

* Certainty equivalent (CE) defined by:

\[
\frac{u(\mu_t(x_{t+1}))}{\text{utility of CE}} = \frac{E_t u(x_{t+1})}{\text{expectation of utility of gamble}} - \beta E_t \left[ u(\delta \mu_t(x_{t+1})) - u(x_{t+1}) I(x_{t+1} \leq \delta \mu_t(x_{t+1})) \right],
\]

where \( u(x) = x^\alpha / \alpha. \)

• For \( \beta > 0 \), if the outcome \( x_{t+1} \) is 'disappointing', i.e. \( x_{t+1} \leq \delta \mu_t(x_{t+1}) \), the utility of the CE is reduced by the penalty function.

• \( \beta \) is the weight attached to this penalty function – more positive \( \beta \) means a higher penalty.
• $\delta$ fixes how easily disappointed the agent is relative to the certainty equivalent. Higher $\delta$ means agents are more easily disappointed.
• Relative risk aversion $1 - \alpha$, Elasticity of intertemporal substitution $1 / (1 - \gamma)$. 
3.1 Advantages of this approach

- The disappointment benchmark is determined endogenously — different from loss aversion.
- Introducing new preference parameters — link to standard time-separable preferences and more general recursive preferences made clear.
- Any reverse engineering is far from obvious.
4 Comments and questions

- No closed form solutions – the CE is the solution to fixed point problem.
  - Therefore difficult to see exactly how GDA affects the pricing kernel
    - For zero relative risk aversion ($\alpha = 1$) and perfectly elastic intertemporal substitution ($\gamma = 1$):
      \[
      M_{t+1} = \frac{1}{1 + \rho} \frac{1 + \beta I \left( \frac{R_{t+1}^x}{1 + \rho} < \delta \right)}{1 + \rho \left( \frac{R_{t+1}^x}{1 + \rho} < \delta \mid \mathcal{I}_t \right)},
      \]
      where $R_{t+1}^x$ is the return on the claim to the consumption endowment (to be determined endogenously in equilibrium).
Can we obtain closed-form solutions in continuous-time for specific parameter values?

- Thus obtain expressions for CAPM, stock returns, riskless rate, stock return volatility.
4.1 Outline of possible approach

Stochastic differential utility with aggregator \((W, \mu)\), where

\[
W(c, z) = \left[\left(1 - \frac{1}{1 + \rho}\right)c^\gamma + \frac{1}{1 + \rho}z^\gamma\right]^{1/\gamma}
\]

and \(\mu\) is defined by the fixed point problem:

\[
\int H(x, \mu(p)) \, dp(x) = 0,
\]

where

\[
H(x, y) = u(x) - \beta \{u(\delta y) - u(x)\} I(x \leq \delta y) - u(y).
\]

Note that \(H\) is discontinuous at \(x = \delta y\).
Characterize utility process $U_t$ as solution to:

$$U_t = E_t \int_t^T g(U_s) \, dL_s + W(c_s, V_s) \, ds,$$

where $L_t$ is local time process of $(U_t, \mu(\sim U_t))$, $g$ to be determined by applying Ito’s Lemma to $H(U_t, \mu(\sim U_t))$.

Discontinuity in $H$ (first order risk aversion) → singular control problem.

- See if a closed form solution can be obtained for special cases, such as

  $$\alpha = 1, \rho = 1, \beta > 0, \delta = 1.$$  

  or

  $$\alpha = 1, \rho = 1, \beta > 0, \delta = 0.$$  

- Use asymptotic analysis to obtain local extensions of solutions for special cases.